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Abstract

This paper measures the welfare gains of switching from inflation-targeting to price-level targeting under imperfect credibility. Vestin (2006) shows that when the monetary authority cannot commit to future policy, price-level targeting yields higher welfare than inflation targeting. We revisit this issue by introducing imperfect credibility, which is modeled as gradual adjustment of the private sector's beliefs about the policy change. We find that gains from switching to price-level targeting are small. A welfare loss occurs, if imperfect credibility is highly persistent.

JEL classification: E31, E52

Bank classification: Credibility; Monetary policy framework

Résumé

Les auteurs mesurent les gains de bien-être attendus de l'abandon d'une cible d'inflation au profit d'une cible fondée sur le niveau des prix en contexte de crédibilité imparfaite. Selon Vestin (2006), la poursuite d'une cible de niveau des prix permet d'améliorer le bien-être lorsque l'autorité monétaire ne peut s'engager au sujet de sa politique future. Les auteurs réexaminent la question en posant l'hypothèse d'une crédibilité imparfaite, modélisée sous la forme d'une adaptation progressive des croyances du secteur privé à l'adoption d'une cible de niveau des prix. Si gains il y a, concluent-ils, ceux-ci sont modestes. On observe toutefois un recul du bien-être si le manque de crédibilité persiste longtemps.

Classification JEL : E31; E52

Classification de la Banque : Crédibilité; Cadre de la politique monétaire

1. Introduction

Price stability, normally defined as low and stable inflation, is the primary stated goal of monetary policy for many central banks around the world. Inflation targeting has become a successful way of implementing that goal in a number of countries, such as Canada, Sweden, New Zealand, and the United Kingdom. Under inflation targeting (IT), the central bank is trying to stabilize the inflation rate around some target value. Such policy implies that the price level can drift arbitrarily far away from any predetermined time trend. Recently, price-level targeting (PT) - a policy that stabilizes the price level around a deterministic trend - has been considered as an alternative approach to achieving price stability. While price-level targeting may potentially deliver better outcomes in the long-run¹, the *transition* from inflation to price-level targeting could destabilize inflation expectations. This is founded on the notion that people may doubt the central bank's willingness to consistently follow the new price-level targeting policy regardless of the shocks that hit the economy. As a result, it may take some time for private agents to adjust their inflation expectations in the aftermath of the policy change. In this paper, we quantify the welfare gains of switching from IT to PT, taking *as given* a sluggish adjustment of inflation expectations during the transition period.

Following Kydland and Prescott (1977), Clarida et al.(1999) show that in the absence of commitment technology monetary policy leads to inefficient outcomes. Specifically, a discretionary central bank is unable to commit to the optimal path of future inflation, which effectively makes expected future inflation independent of its current policy. The lack of control over expected future inflation forces the central bank to meet all of its current-period

¹See Duguay (1994), Svensson (1999) and Coulombe (1998) for discussions of desirability of price-level targeting.

objectives by manipulating the interest rate. As a result, the economy experiences a larger amount of policy-induced volatility than would be the case if commitment were possible.

Clarida et al.(1999) point out that the central bank that lacks commitment will stabilize the inflation rate at a constant target. We refer to such policy regime as inflation targeting (IT). Vestin (2006) argues that it is possible to improve upon this no-commitment outcome by modifying the central bank's policy objective.² He demonstrates that a modification of the central bank's objective function, by including a term for the variation in the price level (possibly around a trend), leads to stabilization of the price level and higher social welfare. In some cases it is possible to replicate the first-best, commitment outcome. We refer to this policy regime, with modified loss function, as price-level targeting (PT). Price-level targeting improves the current policy trade-off between inflation variability and output variability through the *expectation channel*. When a shock pushes the current price level above the target, future inflation is expected to be lower than usual in order to revert the price level back to the target. This in turn counteracts the current inflation increase, due to the standard New Keynesian Phillips Curve relationship. In effect, price-level targeting creates an automatic stabilizer working via the expectation channel.

In this paper, we model a one-time policy switch from inflation targeting to price-level targeting, allowing for *imperfect credibility* of the new policy regime. Here, imperfect credibility is the economic agent's belief that the monetary policy might revert back to inflation targeting in the subsequent period. The degree of imperfect credibility of PT regime is mod-

²It is common in the literature on discretionary policy to assume an exogenous loss function that is delegated to the monetary authority as the objective of its decision problem. This creates an inconsistency in the sense that the monetary authority cannot commit to the first-best policy, but can commit to follow the policy induced by some loss function. We follow the literature, realizing this problem. See Svensson (1999) for a discussion of this point.

eled as the probability that private agents assign to a permanent switch of the policy regime back to IT, taking place in the following period.

We first confirm Vestin’s insight that there are net welfare gains from a policy switch to PT as long as full credibility of PT is immediate, meaning private beliefs are immediately consistent with PT. However, imperfect credibility weakens the effectiveness of the expectation channel under PT. Intuitively, if private agents assign positive probability to a policy reversal from PT back to IT, then with the same probability future inflation is independent of the current price level. As a result, the strength of the negative feedback effect of expected future inflation on current inflation is lower. Furthermore, with a weakened expectation channel, the central bank will be overly aggressive in its attempt to stabilize the price level. Consequently, if imperfect credibility is prolonged, the welfare gains get smaller, eventually turning into net welfare losses.

As a second step, we quantify the welfare gains from switching to PT both under perfect and imperfect credibility. Our two key results are: First, we find that even under perfect credibility, the net benefits of the policy switch are small. For the benchmark calibration, the welfare improvement of PT over IT is equivalent to a permanent reduction in the standard deviation of quarterly inflation by about 0.05 percentage points. So the expectation channel of monetary policy, which received so much attention in the recent literature on inflation and price-level targeting under discretion, is likely not so important.³ Second, we show that a persistent lack of credibility (lasting at least 10 quarters) leads to welfare losses under the

³Another noted potential benefit of price-level targeting is that it decreases the probability of a liquidity trap. Adam and Billi (2007) find that the welfare difference between an unrestricted inflation targeting policy (i.e. optimal monetary policy under discretion) and the one that avoids the zero bound on nominal interest rate, is around 0.0075 percent of consumption. Hence, including a zero bound on nominal interest rates is unlikely to affect our results.

new regime. This implies that short periods of imperfect credibility while decreasing welfare gains from PT do not overturn them. These two results are robust for a wide range of values of the policymakers' weight on the output gap stabilization, the persistence of the cost push shocks, and the slope of the Phillips curve.⁴

The paper is organized as follows: Section 2 introduces the model, Section 3 outlines the solution procedure relegating all the details to the appendices, Section 4 discusses the calibration, results and sensitivity analysis, and finally, Section 5 concludes.

2. Model with imperfect credibility

A. Environment

For our analysis we employ a version of Clarida, Gali, Gertler (1999) model. It is representative of a wide class of general equilibrium models with temporary nominal rigidities. The model generates simple monetary policy rules that are robust across a variety of macroeconomic models. It has been widely used for monetary policy analysis, particularly in recent literature on inflation and price-level targeting. This subsection recaps the main elements of the model.

There are four types of agents in the economy: infinitely lived households, final good producers, intermediate good producers, and a central bank.

The representative household maximizes lifetime expected utility subject to a budget constraint. The (log-linearized) first-order conditions of the household's maximization

⁴Yetman (2005) analyzes PT in a model, where private expectations are permanently misaligned due to rule-of-thumb agents. He finds that under those conditions, PT might be welfare dominated by IT. Here we are focusing on temporary credibility problems after a policy change from IT to PT. Our results are consistent with Yetman's in that a highly persistent lack of credibility may lead to net welfare losses under PT, relative to IT.

problem give rise to the following Euler equation:

$$x_t = -\gamma [i_t - E_t \pi_{t+1}] + E_t x_{t+1} + g_t. \quad (1)$$

In (1) x_t is the output gap, defined as the log deviation of actual output from the potential (flexible-price) output, i_t denotes the nominal interest rate, π_{t+1} is the period $t + 1$ log deviation of the inflation rate from its average level π , g_t is a shock to the real interest rate, and E_t represents the expected value conditional on the household's information through period t .

A competitive final good producer aggregates a variety of intermediate goods into the final good. A monopolistically competitive intermediate good producer faces a dynamic problem in which they set output prices to maximize the expected stream of future dividends subject to the demand conditions and Calvo-type timing restriction on price adjustments. The log-linearized first-order conditions lead to the standard New-Keynesian Phillips Curve relation:

$$\pi_t = \beta E_t [\pi_{t+1}] + \kappa x_t + u_t, \quad (2)$$

where β is the discount factor of the households, and $u_t = \rho u_{t-1} + \varepsilon_t$ is a cost-push shock with normally distributed innovations, $\varepsilon_t \sim N(0, \sigma^2)$. The cost-push shock can be interpreted as a time varying wedge between real wages and the marginal rate of substitution between consumption and labor.

Given constraints (1) and (2), the central bank sets the nominal interest rate i_t to minimize a loss function reflecting the policy regime in place. Following Vestin (2006), we

define inflation targeting as the optimal monetary policy under discretion, with the central bank's period loss function specified as :

$$L_t^0 \equiv \frac{1}{2} \left(\pi_t^2 + \lambda^{IT} x_t^2 \right), \quad (3)$$

and λ^{IT} is the weight on the output gap. Similarly, price-level targeting is the optimal monetary policy under discretion with the central bank's period loss function given by

$$L_t^1 \equiv \frac{1}{2} \left(p_t^2 + \lambda^{PT} x_t^2 \right), \quad (4)$$

where p_t is the period t log-deviation of the price level from a deterministic trend (i.e. price-level target), and λ^{PT} is the corresponding weight on the output gap. Output weights λ^{IT} and λ^{PT} are chosen to maximize social welfare

$$-\frac{1}{2} E \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \lambda x_t^2 \right). \quad (5)$$

Benigno and Woodford (2004) showed that under standard assumptions about utility and monetary transactions technology, equation (5) is a quadratic approximation of a representative household's life-time utility function, and the weight λ depends on structural parameters of that function.⁵ Note that, if the benevolent central bank could commit to its future policy, then it would be able to maximize its natural objective - the social loss

⁵In our benchmark simulations the difference in results between the optimal weight on output under inflation targeting, λ^{IT} , and the output weight λ in the social welfare function (5) was negligible. However, if the persistence of cost-push shocks, ρ , is closer to one, the difference starts to matter. To keep our analysis comparable with Vestin (2006), we follow his assumption that $\lambda^{IT} = \lambda$ in most of our simulations, and comment on the assumption in the sensitivity analysis section. Appendix A contains the proof of existence of λ^{PT} .

function (5). Without commitment, the central bank acts under discretion and optimizes current-period objectives (3) or (4), taking the private expectations of the future variables as being beyond its control.⁶

In this paper, we focus on a policy switch from IT to PT under imperfect credibility. For simplicity, prior to period 0 the central bank follows an IT policy. In period 0 the central bank announces a policy regime change from IT to PT that will take effect in period 1. Starting from period $t = 1$, the central bank's objective changes from (3) to (4).⁷ We assume that the credibility of the new regime is imperfect. In periods $t = 0, 1, 2, 3, \dots$ private agents assign some probability weight, $(1 - \phi_t) \in [0, 1]$, to the possibility of a permanent policy switch back to IT, effective in the following period, $t + 1$. Figure 1 shows the timing of events.

Let $s_t = (p_{t-1}, g_t, u_t, \tau_{t-1}, \phi_t)$ represent the state of the economy at the beginning of period t , where τ_{t-1} is the indicator of the period t -policy regime and takes a value of 0 under IT and 1 under PT (that is, $\tau_{t-1} = 0$ if central bank minimizes L_t^0 and $\tau_{t-1} = 1$ if central bank minimizes L_t^1 in period t).⁸ At the beginning of period t all agents are aware of the current state s_t . Then private agents form expectations of the next period's inflation

$$E_t \pi_{t+1} = \phi_t E[\pi_{t+1} | s_t, \tau_t = 1] + (1 - \phi_t) E_t[\pi_{t+1} | s_t, \tau_t = 0] \quad (6)$$

⁶Although the model does not have an inflation bias as in Kydland and Prescott (1977) there is still a time inconsistency problem in this environment, that leads to suboptimality of discretionary policies. See Clarida et al. (1999) for details.

⁷In this paper we abstract from possible welfare implications of a change in the trend (steady-state) inflation π . So, the policy experiment we are considering is a change from IT to PT with the trend inflation rate being unchanged. As Ascari (2004) points out, the value of the trend inflation may matter for a Calvo-type price adjustment model without full indexation of prices. We bypass this problem by assuming that either the trend inflation rate is zero, or there is full indexation of prices to trend inflation. We do that with a view that the model we use applies more broadly than just to Calvo-type model.

⁸Note that τ_{t-1} is the indicator of the policy regime in the period, t . One could equivalently think that private agents learn the current policy regime from the response of the central bank to current shocks, but this raises an issue of simultaneous formation of expectations, policies and current endogenous variables. The timing assumption does not in any way affects our results and is made for expositional convenience.

and next period's output gap

$$E_t x_{t+1} = \phi_t E [x_{t+1} | s_t, \tau_t = 1] + (1 - \phi_t) E [x_{t+1} | s_t, \tau_t = 0]. \quad (7)$$

$E [\pi_{t+1} | s_t, \tau_t = 1]$ refers to the expected inflation in period $t + 1$, conditional on the policy regime in period $t + 1$ being PT, and $E [\pi_{t+1} | s_t, \tau_t = 0]$ is the expected period $t + 1$ inflation, conditional on the policy regime in period $t + 1$ being IT. After that, the central bank sets the current interest rate i_t to minimize its loss function (3) or (4) subject to constraints (1) and (2). Finally, at the end of period t , private agents observe the policy regime that will be in place at the beginning of the next period, τ_t .

B. Evolution of credibility

There are various problems with specifying a concrete sequence of credibility parameters, $\{\phi_t\}_{t=0}^{\infty}$. Most of them are rooted in the fact that the policy experiment we consider in this paper, namely the move from IT to PT, is purely hypothetical. Various dynamics are plausible, one of them being that the central bank sticks to the new policy regime, and ϕ_t converges stochastically and (in some sense) monotonically to one. Furthermore, there is an issue of whether the bank and the private agents observe current (and past) values of ϕ_t , and how easy it is to predict the future values of ϕ_{t+j} . There is of course always a way to impose some additional structure on the model, which endogenizes the law of motion of ϕ_t . In our view, that route has the disadvantage of making credibility dynamics rigid, and model specific. We take a pragmatic standpoint instead, and assume a very flexible set of deterministic laws of motion, in which it is easy to change the speed of convergence of the

credibility parameter to unity.⁹

We consider two scenarios of the response of credibility to a policy change. In both scenarios, at the time of the policy announcement all agents believe that the change will be reversed in the next period, that is, the degree of credibility at time 0 is $\phi_0 = 0$.¹⁰ In subsequent periods credibility increases with time in a deterministic fashion converging to full credibility, $\phi_t = 1$, asymptotically, or in a finite number of periods. Our two scenarios differ in the smoothness of the speed of convergence.¹¹

In the first scenario, the adjustment of credibility is gradual. Here we assume a simple geometric law of motion for ϕ_t :

Scenario 1 $\phi_{t+1} = \phi_t + \alpha(1 - \phi_t),$

where $\phi_0 = 0$. The speed of adjustment for this process is determined by parameter $\alpha \in [0, 1]$.

In the second scenario, the adjustment is a jump. Under this scenario we assume that ϕ_t jumps discontinuously from zero to one in period $T \geq 1$:

Scenario 2 $\phi_t = \begin{cases} 0, & \text{if } t < T \\ 1, & \text{if } t \geq T. \end{cases}$

The speed of adjustment is governed by the time of the jump, T .

The first scenario may be thought of as an approximation to various gradual patterns of adjustment (stochastic or deterministic), while the second scenario is an approximation to

⁹This simplifying assumption is not uncommon in the literature on the effects of monetary policy change, see for example, Almeida and Bonomo (2002). Erceg and Levin (2003), on the other hand, consider a switch to a lower inflation target in the economy, in which credibility evolves endogenously due to agents' ability to filter information about the unobserved inflation target.

¹⁰This assumption is made for convenience and does not affect our results in any substantial way. If instead we allowed for $\phi_0 > 0$ then there would be a small "announcement" effect of the future policy regime on the period 0 inflation and output gap. Note that period 1 is the first period under PT.

¹¹We have experimented with other deterministic, as well as random convergence scenarios and found similar results.

an S-shaped pattern of adjustment¹². As we show below, both scenarios yield very similar results, so we feel confident that for a wide range of (monotonic) laws of motion the welfare implications of imperfect credibility will be similar to those that we find.

C. Discussion of the model

The model utilized in this paper, while being standard, still raises a set of issues, which we formulate as three questions. Answering these questions below allows us to understand the foundations of the model more clearly as well as relate this paper to the existing literature.

What parameter values should we consider?

Woodford (2003) and Benigno and Woodford (2004) have shown that all of the parameters in the constraints (1),(2) and the social loss function (5) can be derived and calibrated from deep parameters of an underlying model. Thus, as a benchmark, we pick the parameters calibrated by Woodford (2003)¹³. This, however, has the drawback of making our results model-specific. To address this concern we conduct an extensive sensitivity analysis with regards to parameters that show up as being important for the welfare results, or for which there is much uncertainty. These parameters are: the weight placed on the output gap, λ , in the social loss function; the elasticity of inflation with respect to changes in the output gap, κ ; and the persistence of the cost-push shocks, ρ .

Are the constraints on the monetary policy implied by the equations (1) and (2), invariant under monetary policy change and imperfect credibility?

¹²For a wide range of technological innovations the pattern of adoption followed an S type pattern, see for example Rogers, *Diffusion of Innovations*, 5ed 1995. So, if one thinks of policy change as an innovation, it might be reasonable to expect a similar pattern.

¹³The same set of parameters was used in other recent studies of monetary policy, such as Adam and Billi (2007) and Schaumburg and Tambalotti (2007).

It is easy to show that, in a sticky price model with Calvo or Taylor type staggered contracts, the log-linearized versions of the Euler equation (1) and the pricing optimality condition (2) are invariant to the policy change and to imperfect credibility. The only thing that changes is that the expectation operators are now broken into two parts, as in (6) and (7). This is a consequence of the certainty equivalence implied by the log-linearization. The differences in the policies and the patterns of credibility may affect welfare through the second and higher order effects of uncertainty on the first moments of endogenous variables. This may affect the welfare rankings and is taken up next.

How reliable are the welfare rankings obtained with the social loss function (5)?

Benigno and Woodford (2004) and Debortoli and Nunes (2006) have shown that one can readily approximate welfare in a sticky price model with a second-order approximation that takes the form of (5). This is despite tax or monopoly power distortions, and more importantly, *independently* of policies, as long as those policies do not imply large deviations from the non-stochastic steady state around which the approximation is taken. The steady state is the constrained optimal steady state, which has all the tax and monopoly distortions incorporated, but assumes full commitment and a timeless perspective.¹⁴ In this paper we restrict the attention to IT and PT policy rules, which in the absence of shocks, have the same steady state as the constrained optimal ones.¹⁵ Also, as it will become clear from our parametrization, the magnitudes of shocks we consider are small. As a result, the welfare rankings of alternative policies, implied by (5), are second-order accurate.

¹⁴See Benigno and Woodford (2004) and Debortoli and Nunes (2006) for details.

¹⁵This is where the assumption of full indexation to trend inflation is helpful.

3. Solving the model

As in Clarida, Gali, Gertler (1999) we can split the problem of the central bank into two parts. First, the central bank chooses the values of the current output gap, x_t , and the current inflation, π_t , that satisfy the Phillips curve constraint (2). Second, it sets the interest rate, i_t , to satisfy the constraint (1) with the chosen value of the output gap, x_t . This dissection of the problem allows us to ignore the constraint (1) altogether and assume that the central bank can directly set the output gap, x_t . It also implies that we can further ignore the interest rate shocks, g_t , and suppress them in the state space representation. So, let $s_t = (p_{t-1}, u_t, \tau_{t-1}, \phi_t)$ represent the state of the economy at the beginning of period t , and $s^t = (s_t, s^{t-1})$ be the history of the economy at the beginning of period t . With this notation set, we are ready to analyze the problem of the central bank under IT and PT.

A. Inflation targeting

The inflation targeting policy is a solution to the following problem of the central bank

$$V^{IT}(s^t) = \min_{x_t} \left[\frac{1}{2} (\pi_t^2 + \lambda x_t^2) + \beta E_t [V^{IT}(s^{t+1})] \right]$$

subject to

$$\pi_t = \beta E_t [\pi_{t+1}] + \kappa x_t + u_t$$

$$u_t = \rho u_{t-1} + \varepsilon_t$$

It is straightforward to find the time-stationary solution of this problem.¹⁶ It is

$$x_t = -\frac{\kappa}{\lambda} du_t$$

$$\pi_t = du_t$$

where

$$d = \frac{\lambda}{\kappa^2 + \lambda(1 - \beta\rho)}.$$

Under imperfect credibility of PT, agents put a positive probability weight on the possibility of a *permanent* policy regime switch back to IT. From the above optimal policy rules it is easy to evaluate the expectation of future inflation conditional on the switch $E[\pi_{t+1}|s_t, \tau_t = 0] = d\rho u_t$.

B. Price-level targeting

Under the price-level targeting regime, the problem is more complicated. The central bank's problem under PT is

$$V(s^t) = \min_{x_t} \left[\frac{1}{2} (p_t^2 + \lambda^{PT} x_t^2) + \beta E_t V(s^{t+1}) \right] \quad (8)$$

subject to

$$\pi_t = \phi_t E[\pi_{t+1}|s_t, \tau_t = 1] + (1 - \phi_t) E_t[\pi_{t+1}|s_t, \tau_t = 0] + \kappa x_t + u_t$$

$$u_t = \rho u_{t-1} + \varepsilon_t$$

¹⁶For details see Clarida, Gali and Gertler (1999).

$$\phi_{t+1} = f(\phi_t)$$

where $f(\phi_t)$ is the law of motion of the credibility parameter, and λ^{PT} is the weight on the output gap that minimizes the social loss (5) in a fully credible price-level targeting regime (i.e. with $\phi_t = 1$ for all $t = 1, 2, 3, \dots$).

Appendix A shows how to find the value of λ^{PT} numerically, and Appendix B proves that the solution of the problem (8) takes the following form:

$$p_t = a(\phi_t)p_{t-1} + b(\phi_t)u_t \quad (9)$$

$$x_t = -c(\phi_t)p_{t-1} - d(\phi_t)u_t,$$

$$\pi_t = p_t - p_{t-1} \quad (10)$$

where the coefficients $a(\phi_t)$ and $b(\phi_t)$ solve

$$a(\phi_t) = \frac{\lambda^{PT} \left(1 + \phi_t \beta \left(1 - a(\phi_{t+1})\right)\right)}{\Gamma(\phi_t, \phi_{t+1}, \phi_{t+2})} \quad (11)$$

$$b(\phi_t) = \frac{\lambda^{PT} \left(1 + (1 - \phi_t)\beta d\rho + \beta\rho\phi_t b(\phi_{t+1})\right) \left(1 + \phi_t \beta \left(1 - a(\phi_{t+1})\right)\right)}{\Gamma(\phi_t, \phi_{t+1}, \phi_{t+2})} \\ - \frac{\beta\rho\lambda^{PT} \left[1 + (1 - \phi_{t+1})\beta d\rho + \beta\rho\phi_{t+1}b(\phi_{t+2}) - b(\phi_{t+1}) - \beta\phi_{t+1} \left(1 - a(\phi_{t+2})\right) b(\phi_{t+1})\right]}{\Gamma(\phi_t, \phi_{t+1}, \phi_{t+2})} \quad (12)$$

with the denominator

$$\Gamma(\phi_t, \phi_{t+1}, \phi_{t+2}) = \kappa^2 + \lambda^{PT} \left(1 + \phi_t \beta \left(1 - a(\phi_{t+1})\right)\right)^2$$

$$+\beta\lambda^{PT} \left[1 - a(\phi_{t+1}) - \beta \left\{ \phi_{t+1} \left(1 - a(\phi_{t+2}) \right) a(\phi_{t+1}) \right\} \right].$$

The coefficients $c(\phi_t)$ and $d(\phi_t)$ can be determined by substituting the value of p_t from (9) into the formula for output gap

$$x_t = -\frac{1}{\kappa}p_{t-1} + \frac{1}{\kappa} (1 + \phi_t\beta (1 - a_{t+1})) p_t - \frac{1 + (1 - \phi_t)\beta d\rho + \beta\rho\phi_t b_{t+1}}{\kappa} u_t.$$

Thus, given a deterministic sequence $\{\phi_t\}_{t=1}^{\infty}$, and a stochastic sequence $\{u_t\}_{t=1}^{\infty}$, we can solve the model for time paths of output gap and inflation.¹⁷

C. Welfare measure

The expected social loss function (5) is our welfare measure. Since it involves unconditional expectations, the infinite sum in (5) must be integrated over all possible paths of the cost-push shocks $\{u_t\}_{t=0}^{\infty}$. The integration is relatively easy to accomplish analytically for stationary paths of the output gap and inflation. However, the policy experiment in our paper implies non-stationary dynamics, so we must evaluate the unconditional expectation in (5) for every given path of credibility, $\{\phi_t\}_{t=0}^{\infty}$, that we want to consider. Alternatively, we can evaluate the approximate value of the social welfare (5) by taking a simple average of the realized ex-post losses, generated from a large number of random sequences $\{u_t\}_{t=1}^T$. We do that over 1000 random sequences of 3000 periods each, $\{u_t\}_{t=1}^{3000}$.

Once we have different values of the social welfare (5), implied by different paths of

¹⁷For the gradual adjustment scenario (scenario 1) we use a projection method to solve two functional equations (11)-(12) for two (approximate) functions $a(\phi)$ and $b(\phi)$, given the law of motion for ϕ . For the jump adjustment scenario (scenario 2) we can simply solve equations (11)-(12) backward, starting from the period T , in which we know $\phi_T = \phi_{T+1} = \phi_{T+2} = \dots = 1$.

credibility $\{\phi_t\}_{t=0}^{\infty}$, we need to compare them using some tractable welfare measure. We introduce such a welfare measure for stationary dynamics first, and then extend it to a non-stationary case.

Suppose \mathcal{L}^{IT} is the value of the social welfare loss (5) implied by the perpetual inflation targeting (IT) policy. This is as if PT has never been introduced in the first place. Next, suppose \mathcal{L}^{PT} is the value of the social welfare loss (5) implied by the perpetual price-level targeting (PT) policy. It is as if a fully credible PT has been in place in period 0 and after. In both cases, the dynamics are stationary, so we can easily evaluate the expected infinite sums in (5):

$$\begin{aligned}\mathcal{L}^{IT} &= \frac{1}{2}E \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda x_t^2) = \frac{[St.Dev.(\pi_t^{IT})]^2 + \lambda [St.Dev.(x_t^{IT})]^2}{2(1-\beta)} \\ \mathcal{L}^{PT} &= \frac{1}{2}E \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda x_t^2) = \frac{[St.Dev.(\pi_t^{PT})]^2 + \lambda [St.Dev.(x_t^{PT})]^2}{2(1-\beta)}.\end{aligned}$$

The right-most expressions in each line above, make it clear, that we can represent the welfare attained under each stationary policy rule as a point on the plane, with the standard deviation of inflation on one axis, and the standard deviation of output gap on the other axis. Figure 2 shows these two points for IT and PT. It has the standard deviation of the output gap on the horizontal axis and the standard deviation of inflation on the vertical. Given the quadratic period loss function, each level of welfare on this plane is represented by the positive quadrant section of an ellipse. The closer the level curve is to the origin, the higher is the implied welfare (the lower is the social loss). So in Figure 2 a stationary PT regime

implies a higher welfare than a stationary IT regime. We measure the welfare difference between the two points (corresponding to PT and IT) as the vertical distance between their level curves, evaluated along the vertical axis, as shown in Figure 2. Note that the units of measurement along the vertical axis are in terms of the equivalent standard deviation of inflation that would give the same social loss as a policy in question. In other words, we evaluate the welfare difference between two policies as an *equivalent permanent reduction in the standard deviation of inflation* that would make the social loss under IT equal to that under PT. Thus the welfare difference between IT and PT, in our metric, is measured (in percentage points) as

$$\Delta = 100 \left[\sqrt{2(1-\beta) \mathcal{L}^{IT}} - \sqrt{2(1-\beta) \mathcal{L}^{PT}} \right].$$

It is now easy to generalize the welfare metric to non-stationary dynamics, implied by the gradual adjustment of credibility. Let $\mathcal{L}^{Grad}(\alpha)$ be the value of the expected loss function, $\frac{1}{2}E \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda x_t^2)$, that is achieved when credibility evolves according to $\phi_{t+1} = \phi_t + \alpha(1 - \phi_t)$. Then $\mathcal{L}^{Grad}(1)$ is the immediate full credibility benchmark.¹⁸ We report

$$\Delta(\alpha) = 100 \left[\sqrt{2(1-\beta) \mathcal{L}^{Grad}(\alpha)} - \sqrt{2(1-\beta) \mathcal{L}^{Grad}(1)} \right] \quad (13)$$

for different values of α , as the welfare losses due to various degrees of imperfect credibility parametrized by α .

Similarly, let $\mathcal{L}^{Jump}(T)$ be the value of the expected loss function, $\frac{1}{2}E \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda x_t^2)$,

¹⁸Remember that $\phi_0 = 0$ for all cases that we consider.

that is achieved when credibility evolves according to

$$\phi_t = \begin{cases} 0, & \text{if } t < T \\ 1, & \text{if } t \geq T. \end{cases} .$$

Then $\mathcal{L}^{Jump}(1)$ is the immediate full credibility benchmark.¹⁹ We report

$$\Delta(T) = 100 \left[\sqrt{2(1-\beta)\mathcal{L}^{Jump}(T)} - \sqrt{2(1-\beta)\mathcal{L}^{Jump}(1)} \right] \quad (14)$$

for different values of T , as the welfare losses due to various degrees of imperfect credibility parametrized by T .

The welfare metric introduced above has a number of advantages: 1) it allows welfare gains (or losses) from the policy switch to be compared directly with the actual standard deviation of inflation, observed in the data; 2) it makes our welfare comparisons less sensitive to the variation in the welfare weight on output gap, λ , of which there is much uncertainty; 3) as was shown above, it is well suited for comparing welfare under non-stationary policy rules.

An alternative welfare metric that is often used is a steady state consumption equivalent compensation. We did not follow that path, because using consumption equivalents would make our results much more model specific. We choose the standard deviation of inflation as our metric because it allows for a direct comparison of welfare magnitudes across a wide set of models with nominal rigidities and with the standard deviation of inflation in the data.

¹⁹Observe that $\mathcal{L}^{Grad}(1) = \mathcal{L}^{Jump}(1)$, since both patterns of adjustment imply the same time path of ϕ_t .

4. Parametrization and results

A. Benchmark

As a benchmark set of preference parameters we use the values from Woodford (2003, Table 6.1)

$$\beta = 0.99$$

$$\lambda = 0.048$$

$$\kappa = 0.024.$$

We set the benchmark persistence of the cost push shocks at $\rho = 0.48$, halfway between the estimates of Adam and Billi (2005), $\rho = 0$, and of Ireland (2004), $\rho = 0.96$. As Adam and Billi (2005) note, the difference between these two estimates seems to be driven by the different corresponding sample lengths. Given this high degree of uncertainty about the persistence parameter, we choose a midpoint value and carry out an extensive sensitivity analysis later. We do the same for other coefficients for which there is much uncertainty. These are λ and κ . Finally, the standard deviation of the cost-push shocks is pinned down by the standard deviation of inflation in the model under inflation targeting:

$$st.dev. (\pi_t) = \frac{\lambda}{\kappa^2 + \lambda(1 - \beta\rho)} \frac{\sigma}{\sqrt{1 - \rho^2}}.$$

Standard deviation of quarterly CPI inflation rate in Canada during the inflation targeting period (from 1992:Q1 to 2007:Q2) was 0.4 percentage points²⁰. Hence the standard

²⁰The estimated standard deviation of inflation is practically unchanged if we take a later period, e.g. 1996:1-2007:2, after the inflation target in Canada was gradually reduced to its current value of 2 percent

deviation of the cost-push shocks in the model is

$$\sigma = \frac{\kappa^2 + \lambda(1 - \beta\rho)}{\lambda} \sqrt{1 - \rho^2} \cdot 0.004.$$

Figure 3 reports the welfare results for the benchmark set of parameters under the gradual patterns of adjustment in credibility (Scenario 1). The solid horizontal line shows the welfare loss of a fully credible IT regime relative to a fully credible PT regime (i.e. $\Delta^{Grad}(IT) = 100 \left[\sqrt{2(1 - \beta) \mathcal{L}^{IT}} - \sqrt{2(1 - \beta) \mathcal{L}^{Grad}(1)} \right]$), measured as the equivalent permanent change in the standard deviation of inflation, in percentage points. As we can see, the welfare difference is 0.045 percentage points, or roughly one-tenth of the standard deviation of quarterly inflation from 1992 to 2007. So even under immediate perfect credibility, PT gives only a small welfare gain over IT. Points on the dashed curve in Figure 3 show welfare losses of imperfectly credible PT regimes, with various speeds of adjustment of the credibility parameter ϕ_t , i.e. for various values of α in the gradual law of motion $\phi_t = \phi_{t-1} + \alpha (1 - \phi_{t-1})$. More specifically, the horizontal axis measures how much time it takes for the probability weight on PT, ϕ_t , to reach 0.5. We refer to this time as the “half-time”. From the gradual adjustment formula $\phi_t = \phi_{t-1} + \alpha (1 - \phi_{t-1})$ (starting with $\phi_0 = 0$) the “half-time” is $T^h = \frac{\ln 0.5}{\ln(1-\alpha)}$. Thus, the left end of the dashed curve shows that for the case of rapid adjustment of credibility (high α , or equivalently, low T^h), there is a net welfare gain from the policy change from IT to PT equal to the vertical distance between the solid line and the dashed curve. On the other hand, the right end of the dashed curve shows that, in the case of slow adjustment of credibility (low α , or equivalently, high T^h), there is a net welfare loss from the policy change

annual rate.

from IT to PT, equal to the vertical distance between the dashed curve and the solid line. The break-even point happens at T^h , roughly twelve quarters after the policy change.

Similarly, Figure 4 reports the welfare results for the benchmark set of parameters under the jump-like adjustment in credibility (Scenario 2). The solid horizontal line again shows the welfare loss of an IT regime relative to a fully credible PT regime (i.e. $\Delta^{Jump}(IT) = 100 \left[\sqrt{2(1-\beta)\mathcal{L}^{IT}} - \sqrt{2(1-\beta)\mathcal{L}^{Jump}(1)} \right]$). By construction, of course, $\Delta^{Jump}(IT) = \Delta^{Grad}(IT)$. Points on the dashed curve in the Figure 4 show welfare losses of imperfectly credible PT regimes, with various timings of the jump in the credibility parameter ϕ_t , i.e. with various values of T in the law of motion

$$\phi_t = \begin{cases} 0, & \text{if } t < T, \\ 1, & \text{if } t \geq T. \end{cases}$$

As before, the left end of the dashed curve shows that, in the case of rapid adjustment of credibility (low T), there is a net welfare gain from the policy change from IT to PT equal to the vertical distance between the solid line and the dashed curve. The right end of the dashed curve shows that, in the case of slow adjustment of credibility (high T), there is a net welfare loss from the policy change from IT to PT equal to the vertical distance between the dashed curve and the solid line. The break-even point happens at T between twenty and twenty one quarters after the policy switch.

We derive two main conclusions from these benchmark experiments:

1. Even under immediate full credibility of the PT regime, the welfare gains from the policy change are small.
2. Under both gradual and jump adjustments in credibility it takes more than ten

quarters of imperfect credibility for the policy change to become a welfare-reducing event.

B. Sensitivity analysis

In this section, we present the sensitivity analysis of our results to the variation in the following three parameters: the persistence of cost-push shocks, ρ , the loss function weight on the output gap, λ , and the slope of the Phillips curve, κ . We vary each of these parameters individually, holding all other parameters at their benchmark values. The exception is the standard deviation of the cost-push shocks, σ . As before, we always recalibrate the standard deviation of cost-push shocks to match the volatility of inflation rate in Canada, under IT. The ranges for the parameters are in line with what is used in the literature.

Table 1 (see Section 7) shows the sensitivity analysis for our first result regarding the magnitude of the welfare difference between IT and a perfectly credible PT.²¹ The second row of this table shows the range of the welfare difference between IT and a perfectly credible PT, as the persistence of cost-push shocks, ρ , varies from 0 to 0.96. The welfare difference is increasing in ρ , from 0.02 percentage points (of the equivalent permanent reduction in the standard deviation of inflation) to 0.23 percentage points. It is growing at an increasing rate as ρ approaches one. For example, it reaches 0.1 percentage points when $\rho = 0.8$. So for a large range of values of ρ the welfare difference is quite small.²² The welfare difference is increasing in ρ , because the expectation channel gains in importance as the shocks become more serially correlated. Specifically for a high persistence of shocks, a current shock implies a highly persistent (expected) effect for future inflation. As a result, PT becomes more effective

²¹That is, $\Delta(IT) = 100 \left[\sqrt{2(1-\beta)} \mathcal{L}^{IT} - \sqrt{2(1-\beta)} \mathcal{L}^{Grad}(1) \right]$.

²²Furthermore, at high values of ρ our assumption that $\lambda^{IT} = \lambda$ starts to matter. If instead we chose λ^{IT} optimally, then the maximum welfare difference between such an “optimal” IT and a perfectly credible PT is 0.08 percentage points. See Clarida et al. (1999) for the discussion of the “optimal” IT rules.

at stabilizing the economy via the expectation channel since prices do not move as much as under IT.

The third row of Table 1 shows that the welfare difference between IT and a perfectly credible PT falls from 0.07 to 0.02 percentage points as the loss function's weight on the output gap, λ , increases from 0.012 to 0.2. With a larger weight on the output gap, the central bank is less tolerant to its fluctuations and thus manipulates the output gap less aggressively. Under PT the expected aggressive future response to current inflation shocks is precisely what makes the expectation stabilization channel effective. A less aggressive response, due to higher λ , reduces the effectiveness of the expectation stabilization channel, and thus diminishes the advantage of PT over IT.

The last row of Table 1 shows the corresponding range for the welfare difference, as κ increases from 0.006 to 0.08. The difference is increasing in κ , and ranges between 0.01 and 0.09 percentage points. A higher value of κ makes it easier for the central bank to control inflation via changes in the output gap. As a result, the central bank becomes more aggressive. Under IT this leads to higher volatility costs of discretionary policy, while under PT this increase in the aggressiveness of the monetary policy response to inflation shocks makes the expectation channel more effective. Both of these effects increase the welfare difference between IT and a perfectly credible PT.

Table 2 (see Section 7) summarizes sensitivity results for the break-even number of quarters. The break-even number of quarters depends on two things: the welfare difference between an IT regime and a perfectly credible PT regime (i.e. the distance from the horizontal axis to the solid line in Figures 3 and 4), and the speed with which imperfect credibility raises the transition costs of PT (i.e. the slope of dashed curves in Figures 3 and 4). Since

changes in the model's parameters affects both at the same time, the relationship between each of the three parameters, ρ , λ , κ and the break-even number of quarters is in general, non-monotonic.

The bottom line for Table 2 is that for a wide range of parameters, it takes more than ten quarters of low credibility to make PT worse than IT. Perhaps this is not surprising given that the temporary transition costs are being offset by the long-run (albeit small) benefit from PT. With a discount rate of $\beta = 0.99$, the future benefits of PT weigh heavy.

5. Conclusion

When the monetary authority cannot fully commit to its future actions, price-level targeting provides a stabilization device by linking current policy actions to future inflation expectations, and improves the inflation-output trade-off through its effect on the current price level. While this property may render price-level targeting a desirable policy in the long run, the transition to a new policy regime may destabilize inflation expectations for a long period of time. We ask whether a change from inflation to price-level targeting is still beneficial, taking a sluggish adjustment of private agents' beliefs along the transition path into account. From our quantitative analysis of imperfect credibility, we derive two main conclusions: First, even when a policy change from inflation targeting to price-level targeting is fully credible, the welfare gain from better-anchored inflation expectations under price-level targeting appear to be small. Second, for a wide range of parameters, it takes at least ten or more quarters of imperfect credibility for the net benefits of the policy change to become negative.

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6. Appendices

A. Computing the welfare maximizing weight λ^{PT}

Optimal policy rules under discretion take a similar form for both IT and the fully credible PT regimes

$$p_t = ap_{t-1} + bu_t$$

$$x_t = -cp_{t-1} - du_t.$$

Vestin (2006) shows that the unconditional variances of inflation and output gap are then

$$\begin{aligned} \text{var}(\pi_t) &= e^2 \sigma_u^2 = e^2 \frac{\sigma^2}{1 - \rho^2} \\ \text{var}(x_t) &= h^2 \sigma_u^2 = h^2 \frac{\sigma^2}{1 - \rho^2}, \end{aligned}$$

where

$$\begin{aligned} e^2 &= \frac{2b^2(1 - \rho)}{(1 - a\rho)(1 + a)} \\ h^2 &= \frac{b^2c^2(1 + a\rho) + d^2(1 - a^2)(1 - a\rho) + 2\rho bcd(1 - a^2)}{(1 - a^2)(1 - a\rho)}. \end{aligned}$$

The expected social loss can be found as

$$\frac{1}{2} E \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda x_t^2) = \frac{1}{2} \sum_{t=0}^{\infty} \beta^t (\text{var}(\pi_t) + \lambda \text{var}(x_t)) = \frac{\text{var}(\pi_t) + \lambda \text{var}(x_t)}{2(1 - \beta)}.$$

Now, in Appendix C we show that in a fully credible PT regime the coefficients a, b, c and d depend on the output gap weight, λ^{PT} , in the price-level targeting loss function

$L_t^1 \equiv \frac{1}{2} (p_t^2 + \lambda^{PT} x_t^2)$. Experimenting with different parameter values we confirmed Vestin's finding that the expected loss function as a function of λ^{PT} has a unique minimum for some $\lambda^{PT} > \lambda$. In our codes we use matlab optimization routines to find the optimal λ^{PT} which we then use as a weight on the output gap variability under the Price-level targeting regime.

B. Solving for equilibrium under imperfectly credible PT

For convenience we repeat the problem of the central bank under imperfectly credible PT here

$$V(s^t) = \min_{x_t} \left[\frac{1}{2} (p_t^2 + \lambda^{PT} x_t^2) + \beta E_t V(s^{t+1}) \right]$$

subject to

$$\pi_t = \phi_t E [\pi_{t+1} | s_t, \tau_t = 1] + (1 - \phi_t) d\rho u_t + \kappa x_t + u_t$$

$$u_t = \rho u_{t-1} + \varepsilon_t$$

$$\phi_{t+1} = f(\phi_t)$$

We use the same procedure as in Vestin (2006) to solve the model. Rewrite the Phillips curve as

$$p_t - p_{t-1} = \phi_t \beta E_t [p_{t+1} - p_t] + \kappa x_t + (1 + (1 - \phi_t) \beta d\rho) u_t$$

which, solving for x_t , gives

$$x_t = \frac{1}{\kappa} (1 + \phi_t \beta) p_t - \frac{\beta}{\kappa} \phi_t E_t [p_{t+1}] - \frac{1}{\kappa} p_{t-1} - \frac{(1 + (1 - \phi_t) \beta d\rho)}{\kappa} u_t \quad (15)$$

Guess that the state variable p_t follows a linear rule

$$p_t = a(\phi_t)p_{t-1} + b(\phi_t)u_t \quad (16)$$

where $a(\phi_t)$ and $b(\phi_t)$ are parameters that depend on ϕ_t . This implies

$$E_t [p_{t+1}] = p_t E_t [a(\phi_{t+1})] + E_t [b(\phi_{t+1})u_{t+1}].$$

Assuming that ϕ_{t+1} is independent of the cost-push innovation ε_{t+1} we obtain

$$E_t [p_{t+1}] = p_t E_t [a(\phi_{t+1})] + \rho u_t E_t [b(\phi_{t+1})]$$

Denote

$$a_{t+1} = E_t [a(\phi_{t+1})]$$

$$b_{t+1} = E_t [b(\phi_{t+1})]$$

then we can rewrite (15) as

$$\begin{aligned} x_t &= \frac{1}{\kappa} (1 + \phi_t \beta) p_t - \frac{\beta}{\kappa} \phi_t (a_{t+1} p_t + b_{t+1} \rho u_t) - \frac{1}{\kappa} p_{t-1} - \frac{(1 + (1 - \phi_t) \beta d \rho)}{\kappa} u_t \\ &= -\frac{1}{\kappa} p_{t-1} + \frac{1}{\kappa} (1 + \phi_t \beta (1 - a_{t+1})) p_t - \frac{1 + (1 - \phi_t) \beta d \rho + \beta \rho \phi_t b_{t+1}}{\kappa} u_t \end{aligned} \quad (17)$$

which if solved for p_t gives

$$p_t = \frac{\kappa}{1 + \phi_t \beta (1 - a_{t+1})} x_t + \frac{1}{1 + \phi_t \beta (1 - a_{t+1})} p_{t-1} + \frac{1 + (1 - \phi_t) \beta d\rho + \beta \rho \phi_t b_{t+1}}{1 + \phi_t \beta (1 - a_{t+1})} u_t.$$

Thus,

$$\frac{\partial p_t}{\partial x_t} = \frac{\kappa}{1 + \phi_t \beta (1 - a_{t+1})}.$$

The first-order condition for the central bank's problem is

$$0 = E_t \left\{ \frac{\partial p_t}{\partial x_t} p_t + \lambda^{PT} x_t + \beta \frac{\partial V_{t+1}}{\partial p_t} \frac{\partial p_t}{\partial x_t} \right\}.$$

Guessing the expected derivative of the value function as

$$E_t \left[\frac{\partial V_{t+1}}{\partial p_t} \right] = \gamma_{1,t+1} + \gamma_{2,t+1} p_t + \gamma_{3,t+1} \rho u_t$$

the first-order condition becomes

$$\begin{aligned} 0 &= \frac{\kappa p_t}{1 + \phi_t \beta (1 - a_{t+1})} + \lambda^{PT} x_t + \frac{\beta \kappa (\gamma_{1,t+1} + \gamma_{2,t+1} p_t + \gamma_{3,t+1} \rho u_t)}{1 + \phi_t \beta (1 - a_{t+1})} \\ 0 &= \frac{\kappa p_t + \beta \kappa (\gamma_{1,t+1} + \gamma_{2,t+1} p_t + \gamma_{3,t+1} \rho u_t)}{1 + \phi_t \beta (1 - a_{t+1})} \\ &\quad + \frac{\lambda^{PT}}{\kappa} [(1 + \phi_t \beta [1 - a_{t+1}]) p_t - p_{t-1} - (1 + (1 - \phi_t) \beta d\rho + \beta \rho \phi_t b_{t+1}) u_t] \end{aligned}$$

and after rearrangements

$$\left(\frac{\kappa^2 + \lambda^{PT} (1 + \phi_t \beta (1 - a_{t+1}))^2 + \beta \kappa^2 \gamma_{2,t+1}}{\kappa (1 + \phi_t \beta (1 - a_{t+1}))} \right) p_t$$

$$\begin{aligned}
&= -\frac{\beta\kappa\gamma_{1,t+1}}{1 + \phi_t\beta(1 - a_{t+1})} + \frac{\lambda^{PT}}{\kappa}p_{t-1} \\
&\quad + \left[\frac{\lambda^{PT}(1 + (1 - \phi_t)\beta d\rho + \beta\rho\phi_t b_{t+1})(1 + \phi_t\beta(1 - a_{t+1})) - \beta\rho\kappa^2\gamma_{3,t+1}}{\kappa(1 + \phi_t\beta(1 - a_{t+1}))} \right] u_t
\end{aligned}$$

which solving for p_t gives

$$\begin{aligned}
p_t &= \frac{-\beta\kappa^2\gamma_{1,t+1}}{\kappa^2 + \lambda^{PT}(1 + \phi_t\beta(1 - a_{t+1}))^2 + \beta\kappa^2\gamma_{2,t+1}} + \frac{\lambda^{PT}(1 + \phi_t\beta(1 - a_{t+1}))}{\kappa^2 + \lambda^{PT}(1 + \phi_t\beta(1 - a_{t+1}))^2 + \beta\kappa^2\gamma_{2,t+1}}p_{t-1} \\
&\quad + \left[\frac{\lambda^{PT}(1 + (1 - \phi_t)\beta d\rho + \beta\rho\phi_t b_{t+1})(1 + \phi_t\beta(1 - a_{t+1})) - \beta\rho\kappa^2\gamma_{3,t+1}}{\kappa^2 + \lambda^{PT}(1 + \phi_t\beta(1 - a_{t+1}))^2 + \beta\kappa^2\gamma_{2,t+1}} \right] u_t
\end{aligned}$$

which under our assumed solution $p_t = a(\phi_t)p_{t-1} + b(\phi_t)u_t$ implies that $\gamma_{1,t+1} = 0$ and

$$\begin{aligned}
p_t &= \frac{\lambda^{PT}(1 + \phi_t\beta(1 - a_{t+1}))}{\kappa^2 + \lambda^{PT}(1 + \phi_t\beta(1 - a_{t+1}))^2 + \beta\kappa^2\gamma_{2,t+1}}p_{t-1} \\
&\quad + \left[\frac{\lambda^{PT}(1 + (1 - \phi_t)\beta d\rho + \beta\rho\phi_t b_{t+1})(1 + \phi_t\beta(1 - a_{t+1})) - \beta\rho\kappa^2\gamma_{3,t+1}}{\kappa^2 + \lambda^{PT}(1 + \phi_t\beta(1 - a_{t+1}))^2 + \beta\kappa^2\gamma_{2,t+1}} \right] u_t. \quad (18)
\end{aligned}$$

The envelope theorem applied to the central bank's problem gives

$$E_t \left[\frac{\partial V_{t+1}}{\partial p_t} \right] = E_t \left[\frac{\partial V_{t+1}}{\partial x_{t+1}} \frac{\partial x_{t+1}}{\partial p_t} \right] = E_t \left[\lambda^{PT} x_{t+1} \left(-\frac{1}{\kappa} \right) \right]$$

So

$$E_t \left[\frac{\partial V_{t+1}}{\partial p_t} \right] = -\frac{\lambda^{PT}}{\kappa^2} E_t \left[-p_t + \left[1 + \phi_{t+1}\beta(1 - a_{t+2}) \right] p_{t+1} - \left(1 + (1 - \phi_{t+1})\beta d\rho + \beta\rho\phi_{t+1} b_{t+2} \right) u_{t+1} \right]$$

$$\begin{aligned}
&= -\frac{\lambda^{PT}}{\kappa^2} E_t \left[\begin{array}{c} -p_t + [1 + \phi_{t+1}\beta(1 - a_{t+2})] (a(\phi_{t+1})p_t + b(\phi_{t+1})u_{t+1}) \\ - (1 + (1 - \phi_{t+1})\beta d\rho + \beta\rho\phi_{t+1}b_{t+2}) u_{t+1} \end{array} \right] \\
&= \frac{\lambda^{PT}}{\kappa^2} E_t \left[\begin{array}{c} [1 - (1 + \phi_{t+1}\beta(1 - a_{t+2})) a(\phi_{t+1})] p_t \\ + [1 + (1 - \phi_{t+1})\beta d\rho + \beta\rho\phi_{t+1}b_{t+2} - (1 + \phi_{t+1}\beta(1 - a_{t+2})) b(\phi_{t+1})] u_{t+1} \end{array} \right] \\
&= \frac{\lambda^{PT}}{\kappa^2} \left[\begin{array}{c} [1 - E_t \{ (1 + \phi_{t+1}\beta(1 - a_{t+2})) a(\phi_{t+1}) \}] p_t \\ + [1 + (1 - E_t [\phi_{t+1}])\beta d\rho + \beta\rho E_t [\phi_{t+1}b_{t+2}] - E_t \{ (1 + \phi_{t+1}\beta(1 - a_{t+2})) b(\phi_{t+1}) \}] \rho u_t \end{array} \right] \\
&= \frac{\lambda^{PT}}{\kappa^2} [1 - E_t [a(\phi_{t+1})] - \beta E_t \{ \phi_{t+1} (1 - a_{t+2}) a(\phi_{t+1}) \}] p_t \\
&\quad + \frac{\lambda^{PT}\rho}{\kappa^2} [1 + (1 - E_t [\phi_{t+1}])\beta d\rho + \beta\rho E_t [\phi_{t+1}b_{t+2}] - E_t [b(\phi_{t+1})] - \beta E_t \{ \phi_{t+1} (1 - a_{t+2}) b(\phi_{t+1}) \}] u_t \\
&= \frac{\lambda^{PT}}{\kappa^2} [1 - a_{t+1} - \beta E_t \{ \phi_{t+1} (1 - a_{t+2}) a(\phi_{t+1}) \}] p_t \\
&\quad + \frac{\lambda^{PT}\rho}{\kappa^2} [1 + (1 - E_t [\phi_{t+1}])\beta d\rho + \beta\rho E_t [\phi_{t+1}b_{t+2}] - b_{t+1} - \beta E_t \{ \phi_{t+1} (1 - a_{t+2}) b(\phi_{t+1}) \}] u_t
\end{aligned}$$

This has to be equal to

$$E_t \left[\frac{\partial V_{t+1}}{\partial p_t} \right] = \gamma_{1,t+1} + \gamma_{2,t+1}p_t + \gamma_{3,t+1}\rho u_t = \gamma_{2,t+1}p_t + \gamma_{3,t+1}\rho u_t,$$

which implies

$$\begin{aligned}
\gamma_{2,t+1} &= \frac{\lambda^{PT}}{\kappa^2} [1 - a_{t+1} - \beta E_t \{ \phi_{t+1} (1 - a_{t+2}) a(\phi_{t+1}) \}] \\
\gamma_{3,t+1} &= \frac{\lambda^{PT}}{\kappa^2} [1 + (1 - E_t [\phi_{t+1}])\beta d\rho + \beta\rho E_t [\phi_{t+1}b_{t+2}] - b_{t+1} - \beta E_t \{ \phi_{t+1} (1 - a_{t+2}) b(\phi_{t+1}) \}]
\end{aligned}$$

Substituting these values of coefficients $\gamma_{2,t+1}$, and $\gamma_{2,t+1}$ into (18) we finally obtain

$$\begin{aligned}
a(\phi_t) &= \frac{\lambda^{PT} (1 + \phi_t \beta (1 - a_{t+1}))}{\kappa^2 + \lambda^{PT} (1 + \phi_t \beta (1 - a_{t+1}))^2 + \beta \lambda^{PT} [1 - a_{t+1} - \beta E_t \{ \phi_{t+1} (1 - a_{t+2}) a(\phi_{t+1}) \}]} \\
b(\phi_t) &= \frac{\lambda^{PT} (1 + (1 - \phi_t) \beta d\rho + \beta \rho \phi_t b_{t+1}) (1 + \phi_t \beta (1 - a_{t+1}))}{\kappa^2 + \lambda^{PT} (1 + \phi_t \beta (1 - a_{t+1}))^2 + \beta \lambda^{PT} [1 - a_{t+1} - \beta E_t \{ \phi_{t+1} (1 - a_{t+2}) a(\phi_{t+1}) \}]} \\
&\quad - \frac{\beta \rho \lambda^{PT} [1 + (1 - E_t [\phi_{t+1}]) \beta d\rho + \beta \rho E_t [\phi_{t+1} b_{t+2}] - b_{t+1} - \beta E_t \{ \phi_{t+1} (1 - a_{t+2}) b(\phi_{t+1}) \}]}{\kappa^2 + \lambda^{PT} (1 + \phi_t \beta (1 - a_{t+1}))^2 + \beta \lambda^{PT} [1 - a_{t+1} - \beta E_t \{ \phi_{t+1} (1 - a_{t+2}) a(\phi_{t+1}) \}]}
\end{aligned}$$

Note that so far we allowed for stochastic evolution of ϕ_{t+j} . The only restriction we imposed on the distribution of ϕ is that it is independent of the distribution of the (same period) cost-push innovations, ε_{t+j} . Now, suppose the sequence of ϕ_t evolves deterministically as $\phi_{t+1} = g(\phi_t)$. With the deterministic sequence the expectations of ϕ are degenerate so

$$a_t = E_{t-1} [a(\phi_t)] = a(\phi_t)$$

$$b_t = E_{t-1} [b(\phi_t)] = b(\phi_t).$$

and

$$\begin{aligned}
a(\phi_t) &= \frac{\lambda^{PT} (1 + \phi_t \beta (1 - a(\phi_{t+1})))}{\kappa^2 + \lambda^{PT} (1 + \phi_t \beta (1 - a(\phi_{t+1})))^2 + \beta \lambda^{PT} [1 - a(\phi_{t+1}) - \beta \{ \phi_{t+1} (1 - a(\phi_{t+2})) a(\phi_{t+1}) \}]} \\
b(\phi_t) &= \frac{\lambda^{PT} (1 + (1 - \phi_t) \beta d\rho + \beta \rho \phi_t b(\phi_{t+1})) (1 + \phi_t \beta (1 - a(\phi_{t+1})))}{\kappa^2 + \lambda^{PT} (1 + \phi_t \beta (1 - a(\phi_{t+1})))^2 + \beta \lambda^{PT} [1 - a(\phi_{t+1}) - \beta \{ \phi_{t+1} (1 - a(\phi_{t+2})) a(\phi_{t+1}) \}]} \\
&\quad - \frac{\beta \rho \lambda^{PT} [1 + (1 - \phi_{t+1}) \beta d\rho + \beta \rho \phi_{t+1} b(\phi_{t+2}) - b(\phi_{t+1}) - \beta \phi_{t+1} (1 - a(\phi_{t+2})) b(\phi_{t+1})]}{\kappa^2 + \lambda^{PT} (1 + \phi_t \beta (1 - a(\phi_{t+1})))^2 + \beta \lambda^{PT} [1 - a(\phi_{t+1}) - \beta \{ \phi_{t+1} (1 - a(\phi_{t+2})) a(\phi_{t+1}) \}]}
\end{aligned}$$

These are the same equations as in the equations (11) in the text.

C. Fully credible PT benchmark

Suppose $\phi_T = \phi_{T+1} = \phi_{T+2} = \dots = 1$ for sure. Then the equations under (11) imply

$$a_t = \frac{\lambda^{PT} (1 + \beta (1 - a_{t+1}))}{\kappa^2 + \lambda^{PT} (1 + \beta (1 - a_{t+1}))^2 + \beta \lambda^{PT} [1 - a_{t+1} - \beta (1 - a_{t+2}) a_{t+1}]}$$

$$b_t = \frac{\lambda^{PT} (1 + \beta (1 - a_{t+1})) + \beta \rho \lambda^{PT} \{b_{t+1} - (1 + \beta \rho b_{t+2}) + b_{t+1} [\beta (1 - a_{t+1}) + 1 + \beta (1 - a_{t+2})]\}}{\kappa^2 + \lambda^{PT} (1 + \beta (1 - a_{t+1}))^2 + \beta \lambda^{PT} [1 - a_{t+1} - \beta (1 - a_{t+2}) a_{t+1}]}$$

Let's find the stationary solution

$$a = \frac{\lambda^{PT} (1 + \beta (1 - a))}{\kappa^2 + \lambda^{PT} (1 + \beta (1 - a))^2 + \beta \lambda^{PT} [1 - a - \beta (1 - a) a]}$$

$$b = \frac{\lambda^{PT} (1 + \beta \rho b) (1 + \beta (1 - a)) - \beta \rho \lambda^{PT} [1 + \beta \rho b - b - \beta (1 - a) b]}{\kappa^2 + \lambda^{PT} (1 + \beta (1 - a))^2 + \beta \lambda^{PT} [1 - a - \beta (1 - a) a]}$$

After some manipulations with the last equation, we obtain

$$b = \frac{1 + \beta (1 - a) - \beta \rho}{\frac{\kappa^2}{\lambda^{PT}} + (1 + \beta (1 - a))^2 + \beta (1 - a) [1 - \beta a] - \beta \rho [1 + \beta (1 - a) - \beta \rho + 1 + \beta (1 - a)]}$$

where a is a solution to

$$a \left[\frac{\kappa^2}{\lambda^{PT}} + (1 + \beta (1 - a))^2 + \beta (1 - a) [1 - \beta a] \right] = 1 + \beta (1 - a)$$

Now, for the output gap with perfect credibility

$$x_t = -\frac{1}{\kappa} p_{t-1} + \frac{1}{\kappa} (1 + \beta (1 - a)) p_t - \frac{1 + \beta \rho b}{\kappa} u_t$$

$$= -\frac{1}{\kappa} p_{t-1} + \frac{1}{\kappa} (1 + \beta (1 - a)) (a p_{t-1} + b u_t) - \frac{1 + \beta \rho b}{\kappa} u_t$$

$$\begin{aligned}
&= \frac{1}{\kappa} (-1 + a + a\beta(1 - a)) p_{t-1} + \frac{1}{\kappa} [(1 + \beta(1 - a))b - 1 - \beta\rho b] u_t \\
&= -\frac{(1 - a)(1 - a\beta)}{\kappa} p_{t-1} + \frac{b + b\beta(1 - a) - 1 - \beta\rho b}{\kappa} u_t,
\end{aligned}$$

or finally

$$x_t = -\frac{(1 - a)(1 - a\beta)}{\kappa} p_{t-1} - \frac{1 - b[1 + \beta(1 - \rho - a)]}{\kappa} u_t.$$

D. Computation of equilibrium for jump adjustment in credibility

A special attention is needed to non-stationary law of motion of ϕ_t implied by our scenario 2. In particular, assume that $\phi_t = 0$ for $t = 1, 2, \dots, T - 1$, and $\phi_t = 1$ for $t \geq T$. Abusing notation, let $a(\phi_t, \phi_{t+1}, \phi_{t+2})$ and $b(\phi_t, \phi_{t+1}, \phi_{t+2})$ be the optimal coefficients in the period t , given the values $\phi_t, \phi_{t+1}, \phi_{t+2}$. Then we need to solve the following equations:

1) for period $t = T$ we have

$$\begin{aligned}
a(1, 1, 1) &= \frac{\lambda^{PT} (1 + \beta (1 - a(1, 1, 1)))}{\kappa^2 + \lambda^{PT} (1 + \beta (1 - a(1, 1, 1)))^2 + \beta \lambda^{PT} [1 - a(1, 1, 1) - \beta \{(1 - a(1, 1, 1)) a(1, 1, 1)\}]} \\
b(1, 1, 1) &= \frac{\lambda^{PT} (1 + \beta \rho b(1, 1, 1)) (1 + \beta (1 - a(1, 1, 1)))}{\kappa^2 + \lambda^{PT} (1 + \beta (1 - a(1, 1, 1)))^2 + \beta \lambda^{PT} [1 - a(1, 1, 1) - \beta \{(1 - a(1, 1, 1)) a(1, 1, 1)\}]} \\
&+ \frac{-\beta \rho \lambda^{PT} [1 + \beta \rho b(1, 1, 1) - b(1, 1, 1) - \beta (1 - a(1, 1, 1)) b(1, 1, 1)]}{\kappa^2 + \lambda^{PT} (1 + \beta (1 - a(1, 1, 1)))^2 + \beta \lambda^{PT} [1 - a(1, 1, 1) - \beta \{(1 - a(1, 1, 1)) a(1, 1, 1)\}]}
\end{aligned}$$

2) for period $t = T - 1$ we have

$$\begin{aligned}
a(0, 1, 1) &= \frac{\lambda^{PT}}{\kappa^2 + \lambda^{PT} + \beta \lambda^{PT} [1 - a(1, 1, 1) - \beta \{(1 - a(1, 1, 1)) a(1, 1, 1)\}]} \\
b(0, 1, 1) &= \frac{\lambda^{PT} (1 + \beta d \rho)}{\kappa^2 + \lambda^{PT} + \beta \lambda^{PT} [1 - a(1, 1, 1) - \beta \{(1 - a(1, 1, 1)) a(1, 1, 1)\}]} \\
&+ \frac{-\beta \rho \lambda^{PT} [1 + \beta \rho b(1, 1, 1) - b(1, 1, 1) - \beta (1 - a(1, 1, 1)) b(1, 1, 1)]}{\kappa^2 + \lambda^{PT} + \beta \lambda^{PT} [1 - a(1, 1, 1) - \beta \{(1 - a(1, 1, 1)) a(1, 1, 1)\}]}
\end{aligned}$$

3) for period $t = T - 2$ we have

$$a(0, 0, 1) = \frac{\lambda^{PT}}{\kappa^2 + \lambda^{PT} + \beta\lambda^{PT} [1 - a(0, 1, 1)]}$$

$$b(0, 0, 1) = \frac{\lambda^{PT} (1 + \beta d\rho)}{\kappa^2 + \lambda^{PT} + \beta\lambda^{PT} [1 - a(0, 1, 1)]} + \frac{-\beta\rho\lambda^{PT} [1 + \beta d\rho - b(0, 1, 1)]}{\kappa^2 + \lambda^{PT} + \beta\lambda^{PT} [1 - a(0, 1, 1)]}$$

4) for period $t = T - 3$ we have

$$a^1(0, 0, 0) = \frac{\lambda^{PT}}{\kappa^2 + \lambda^{PT} + \beta\lambda^{PT} [1 - a(0, 0, 1)]}$$

$$b^1(0, 0, 0) = \frac{\lambda^{PT} (1 + \beta d\rho)}{\kappa^2 + \lambda^{PT} + \beta\lambda^{PT} [1 - a(0, 0, 1)]} + \frac{-\beta\rho\lambda^{PT} [1 + \beta d\rho - b(0, 0, 1)]}{\kappa^2 + \lambda^{PT} + \beta\lambda^{PT} [1 - a(0, 0, 1)]}$$

4) for period $t = T - 4$ and beyond we have

$$a^{j+1}(0, 0, 0) = \frac{\lambda^{PT}}{\kappa^2 + \lambda^{PT} + \beta\lambda^{PT} [1 - a^j(0, 0, 0)]}$$

$$b^{j+1}(0, 0, 0) = \frac{\lambda^{PT} (1 + \beta d\rho)}{\kappa^2 + \lambda^{PT} + \beta\lambda^{PT} [1 - a^j(0, 0, 0)]} + \frac{-\beta\rho\lambda^{PT} [1 + \beta d\rho - b^j(0, 0, 0)]}{\kappa^2 + \lambda^{PT} + \beta\lambda^{PT} [1 - a^j(0, 0, 0)]}$$

So, from period $t = T - 2$ back we can compute coefficients recursively.

7. Tables

TABLE 1: SENSITIVITY ANALYSIS REGARDING THE WELFARE GAINS
FROM A SWITCH TO PT.

Parameter	Range of parameter	Welfare difference between IT and a perfectly credible PT, percentage points
Persistense of cost push shocks, ρ	0 - 0.96	0.02 - 0.23
Welfare weight on output gap, λ	0.012 - 0.2	0.07 - 0.02
Slope of Phillips curve, κ	0.006 - 0.08	0.01 - 0.09

TABLE 2: SENSITIVITY ANALYSIS REGARDING THE BREAK EVEN PERIOD
AFTER A SWITCH TO PT.

Parameter	Range of parameter	Range for the break-even number of quarters	
		Half-time (gradual adj.)	Jump-period (jump adj.)
Persistense of cost push shocks, ρ	0 - 0.96	10.5 - 11.5	19 - 23
Welfare weight on output gap, λ	0.012 - 0.2	10 - 15	18-31
Slope of Phillips curve, κ	0.006 - 0.08	10 - 19	18-33

8. Figures

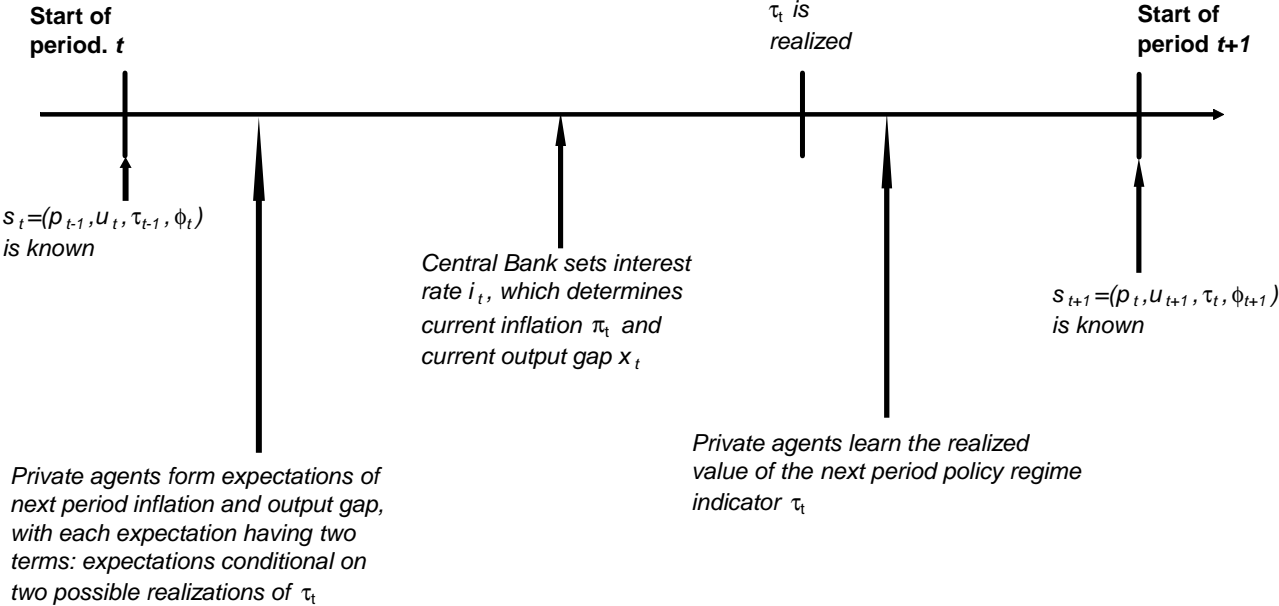


Figure 1: Timing of events

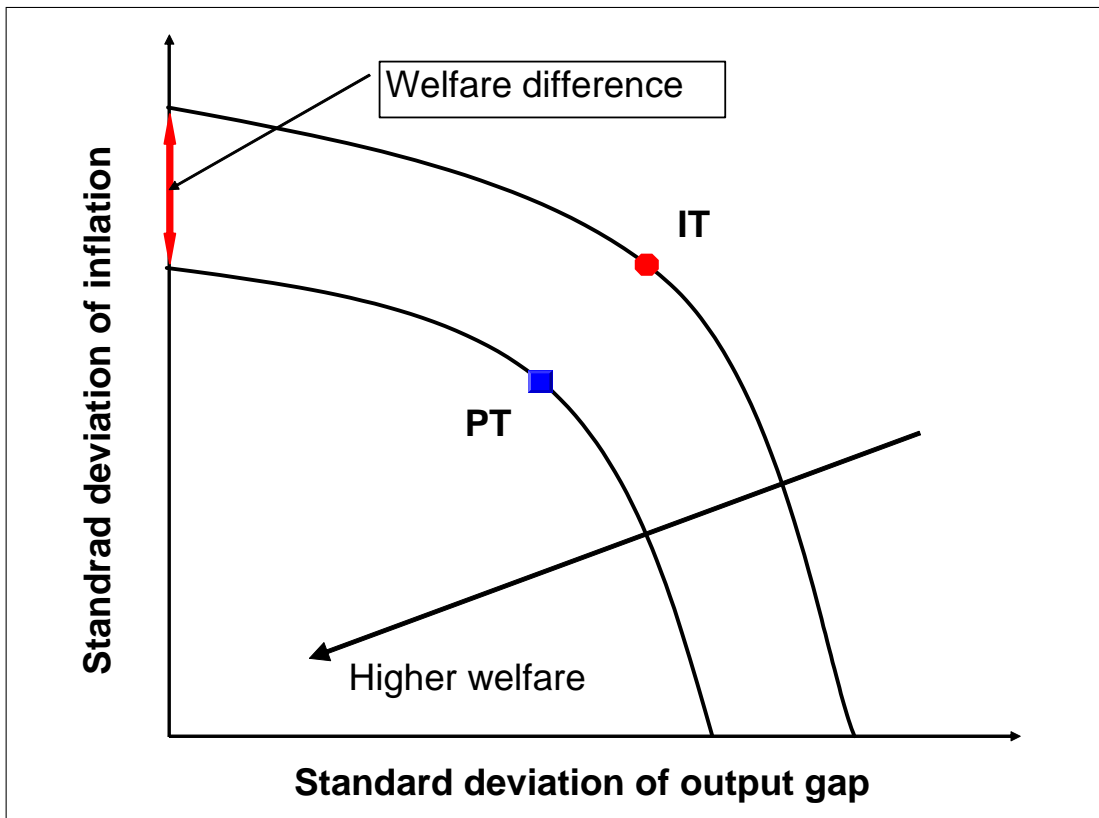


Figure 2: Welfare metric: we use the equivalent difference in the standard deviation of inflation as our measure of welfare difference between two alternative policy regimes.

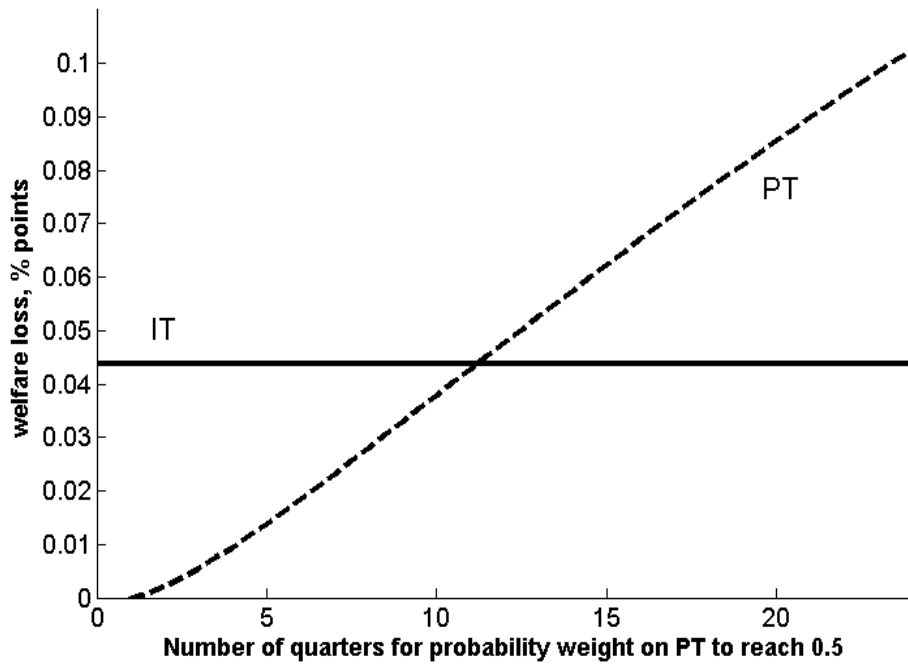


Figure 3: Benchmark results: Welfare losses of various monetary policy regimes minus welfare loss of PT under perfect credibility. Solid line is for IT, dashed curve is for PT with various degrees of imperfect credibility under Scenario 1 (various speeds of gradual adjustment in credibility).

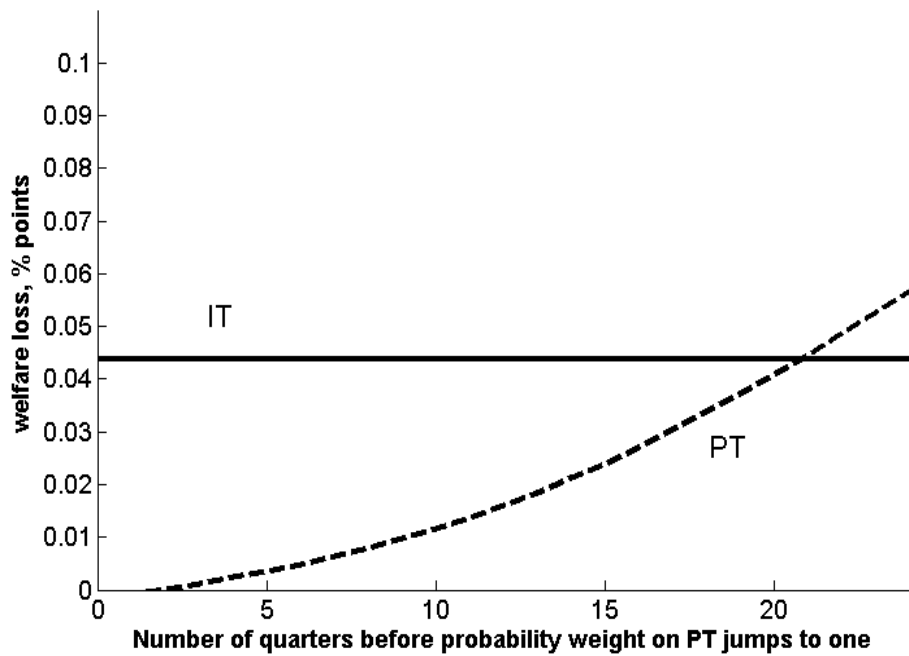


Figure 4: Benchmark results: Welfare losses of various monetary policy regimes minus welfare loss of PT under perfect credibility. Solid line is for IT, dashed curve is for PT with various degrees of imperfect credibility under Scenario 2 (various speeds of jump adjustment in credibility, T).