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**Oleksiy Kryvtsov<sup>1</sup> and Virgiliu Midrigan<sup>2</sup>**

<sup>1</sup>Canadian Economic Analysis Department  
Bank of Canada  
Ottawa, Ontario, Canada K1A 0G9  
okryvtsov@bankofcanada.ca

<sup>2</sup>New York University

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## Abstract

Kryvtsov and Midrigan (2008) study the behavior of inventories in an economy with menu costs, fixed ordering costs and the possibility of stock-outs. This paper extends their analysis to a richer setting that is capable of more closely accounting for the dynamics of the US business cycle. We find that the original conclusion survives in this setting: namely, the model requires an elasticity of real marginal cost to output approximately equal to the inverse intertemporal elasticity of substitution in consumption in order to account for the countercyclicality of the aggregate inventory-to-sales ratio in the data.

*JEL classification: E31, F12*

*Bank classification: Business fluctuations and cycles; Transmission of monetary policy*

## Résumé

Kryvtsov et Midrigan (2008) ont étudié le comportement des stocks dans une économie où il est coûteux de modifier les prix, où la passation de commandes est assortie de coûts fixes et où les entreprises peuvent rater des ventes. Cette fois, les auteurs transposent leur analyse dans un modèle plus évolué capable de rendre compte au plus près de la dynamique du cycle de l'économie américaine. Leur conclusion initiale s'y voit corroborée. Pour rendre compte de l'évolution anticyclique du ratio agrégé stocks/ventes d'après les données, le modèle doit bel et bien intégrer une élasticité du coût marginal réel par rapport à la production qui soit approximativement égale à l'inverse de l'élasticité de substitution intertemporelle de la consommation.

*Classification JEL : E31, F12*

*Classification de la Banque : Cycles et fluctuations économiques; Transmission de la politique monétaire*

# 1. Introduction

Real rigidities are factors that dampen the responsiveness of a firm's desired price to a monetary disturbance. Recent work with New Keynesian sticky price models<sup>1</sup> has argued that real rigidities are a key ingredient necessary to reconcile the apparently slow response of prices to nominal disturbances at the aggregate level<sup>2</sup> with the fairly rapid rate at which individual price setters update their nominal prices<sup>3</sup>.

Models with real rigidities can be broadly categorized into two classes.<sup>4</sup> The first class of models is characterized by assumptions on preferences or technology that make it costly for firms to charge prices that are too different from those of their competitors. Those firms that choose to reset their nominal prices in time of a monetary disturbance thus choose to not fully respond to this disturbance in order to avoid the losses associated with deviating from their competitor's prices.<sup>5</sup> Thus even though prices change frequently in nominal terms, they initially respond little to the monetary injection because of the pricing complementarity arising from non-constant demand elasticities and/or upward sloping marginal cost at the individual producer's level. Although measuring price elasticities or scale returns in the production function is difficult in practice, recent work using micro-price data has argued that simple versions of models that feature this first class of real rigidities are difficult to reconcile with the observed dispersion in relative prices in very narrowly defined product groups within outlets.<sup>6</sup>

In this paper, we focus on a second class of real rigidities that lower the elasticity of economy-wide real marginal cost to output. In this second class of models, assumptions on preferences, the degree to which factor utilization can vary, or frictions in the labor market or in the market for intermediate inputs generate slow adjustment of (nominal) factor prices to a monetary shock. As a result, real marginal costs of production respond little to a monetary

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<sup>1</sup>Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2007) are two well-known examples.

<sup>2</sup>Christiano, Eichenbaum and Evans (1999), Romer and Romer (2004), Friedman (1968).

<sup>3</sup>Bils and Klenow (2005), Klenow and Kryvtsov (2008), Nakamura and Steinsson (2008).

<sup>4</sup>Ball and Romer (1990).

<sup>5</sup>Kimball (1995), Eichenbaum and Fisher (2004), Altig, Christiano, Eichenbaum and Evans (2004).

<sup>6</sup>Klenow and Willis (2006), Dotsey and King (2005), Burstein and Hellwig (2007).

disturbance, thus amplifying real effect of the shock.<sup>7</sup>

Notice that this second class of real rigidities is, in effect, a set of assumptions on aggregate quantities, and in particular, on the firms' (collective) ability to hire additional labor during booms (or hoard labor during recessions), purchase intermediate inputs, and vary capital's workweek. Even when real rigidities take the form of sticky wages or intermediate good's prices, as in much of the recent work, an important assumption made is that these sticky prices are allocative and quantities are demand-determined. These assumptions, that quantities can be relatively costlessly varied during the cycle, and that factor adjustment costs are small, are clearly other key ingredients that are necessary to lower the elasticity of economy-wide marginal cost of production to output.

The discussion above suggests that inferring the elasticity of real marginal cost to output, a measure of the strength of real rigidities in this second class of models, is difficult in practice. In particular, the researcher must be able to measure the relative importance of factor adjustment costs, the degree to which factor prices are allocative, the cost of varying the work-week of capital and labor, as well as the degree of frictions in the labor and intermediate goods market.

Bils and Kahn (2000) show that the behavior of inventories over the cycle is informative about the cyclical nature of costs. In Kryvtsov and Midrigan (2008) we use Bils and Kahn's insights to gauge the implications of models of the second class of real rigidities for the behavior of inventories. If the marginal cost of acquiring and holding inventories is indeed lower in times of monetary expansions, we should see this lower cost reflected not only in a slow adjustment of prices to a monetary shock, but also in an increase in the firm's inventory holdings. In fact, models with inventories predict that a firm's price is proportional to its shadow valuation of its inventories. In turn, when the firm's cost of buying and holding inventories decreases (as it does in times of a monetary expansion), the firm purchases more

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<sup>7</sup>Values of elasticities of real marginal cost to output in models with real rigidities include: 0.15 in Woodford (2002), 0.33 in Dotsey and King (2005), 0.34 in Smets and Wouters (2007). In Chari, Kehoe, McGrattan's (2000) model without real rigidities the elasticity is 2.25.

inventories so as to equalize its shadow valuation of inventories to their marginal cost. Thus real rigidities of this second class must operate through inventories: an increase in the stock of inventories held by the firm is necessary for the shadow valuation of inventories (given concavity of the value function) to decrease and thus for the firm's real price (relative to the money stock) to fall. If the firm is unable to purchase more inventories, either because of the quantity restrictions by suppliers, or because of other costs of adjusting the stock of inventories, the relatively lower factor prices do not translate into a lower shadow valuation of inventories, and the firm finds it optimal to keep its real price high. We thus argue that a model's ability to account for the behavior of inventories in the data (and in particular the strong counter-cyclicality of the inventory-to-sales ratio) is an important empirical test of this class of models.

Our earlier paper studies a menu cost sticky price model in which firms hold inventories of goods from one period to another in order to a) avoid stockouts given a delay between orders and demand uncertainty, and b) economize on fixed ordering costs. The model is sufficiently rich in that it incorporates these two most widely studied inventory motives in recent work, and yet very parsimonious in that only two additional parameters are added to otherwise standard menu cost model (the size of the fixed ordering costs and the volatility of demand shocks). These parameters are calibrated to match several micro-economic moments in the inventory data. We then study the model's responses to monetary disturbances under a number of assumptions regarding the strength of real rigidities (modeled there as a wedge in the consumer's labor-leisure tradeoff).

In that paper we find that even small departures of the elasticity of real marginal cost to consumption away from the inverse of the intertemporal elasticity of substitution (IES) give rise to large differences in the inventory-to-sales ratio's simulated response to a monetary shock. When the elasticity of real marginal cost to consumption is equal to the inverse IES, the cost of purchasing and holding inventories (which depends on both the wholesale price and the real interest rate) does not change with a monetary shock and the firm does not substitute intertemporally. In contrast, when the elasticity of real marginal cost to consumption is lower

than the inverse IES, the combined cost of acquiring and holding inventories decreases and the firm finds it optimal to raise its inventory stock by a large amount. We thus conclude that in that simple setup it is difficult to reconcile strong real rigidities (low elasticities of real marginal cost to output) with the behavior of inventories in the data, unless one also assumes a high elasticity of intertemporal substitution.

In this paper we extend the analysis in Kryvtsov and Midrigan (2008) along several dimensions in order to gauge the robustness of those results. In particular, highly non-linear nature of firm decision rules in our earlier paper precluded us from embedding our microeconomic model of inventories into a full-blown medium-scale equilibrium model of the type studied in Christiano, Eichenbaum and Evans (2005) or Smets and Wouters (2007). One objection to our earlier analysis is that the simplicity of our original setup precludes it from accounting for the dynamic responses of output, interest rates, costs and inflation to the monetary disturbance. Given that the dynamic paths of interest rates, inflation and costs are crucial for firm's optimal inventory holdings, the concern is that our earlier results are in part driven by our model's simplicity and inability to match the dynamics of key macroeconomic variables in the data. A second concern that arises is that our results stem from the non-linear firm policy rules in our earlier setup with fixed price and inventory adjustment costs. Finally, we are able to investigate whether perturbations of the model (allowing for higher depreciation rates and adding adjustment costs on factors of production) help alleviate the counterfactual implications of models with inventories and real rigidities.

To address these concerns, we study in this paper a Calvo sticky price model in which firms adjust nominal prices with a constant hazard and in which demand uncertainty and a lag between orders and deliveries of goods give rise to a stockout-avoidance motive that makes it optimal for firms to carry inventories across periods. To build intuition for the mechanism at work, we first start with a simple cash-in-advance version of the model that is very similar to that in our earlier paper. We show that our earlier results are robust in this alternative setup. In particular, deviations of the elasticity of real marginal cost to consumption from the inverse IES give rise to a sharp increase in the inventory-sales ratio during monetary



expansions. Given that one component of output is inventory investment, this large spike in the inventory-sales ratio in models with strong real rigidities implies a counterfactually large spike in output (almost ten times larger than the increase in consumption after the monetary expansion). Further more, allowing higher rates of depreciation (8% per month) does resolve this counterfactual implication of the model but now provides counterfactual microeconomic implications. In particular, with such high depreciation rates, firms hold the 1.4 monthly inventory-to-sales ratio observed in the US data only if faced with considerable demand uncertainty (237% standard deviation of shocks when the elasticity of substitution is equal to 5). Similarly, adding convex adjustment costs does help bring the response of inventories to a monetary shock in line with the data; however, they do so by raising firm's costs of replenishing its stock of inventories. In this case, relatively low factor prices during booms do not feed into lower retail prices as the firm's effective marginal cost of purchasing more inventories increases because of the adjustment costs. Finally, we ask in this simple setup: what are the consequences of allowing wedges in a consumer's savings-consumption and consumption-leisure decision that allow the model to *exactly* match the impulse responses of wages and real interest rates to a monetary policy shock. We find that absent additional frictions on the firm's ability to purchase inventories, the inventory-sales ratio increases strongly during a boom, as in the simpler versions of the model without wedges.

In a final set of exercises we embed the stockout-avoidance inventory holding motive into a medium-scale macroeconomic model with a richer set of shocks and frictions of the type studied in Smets and Wouters (2007). We find that our original results extend to this setting. In particular, a version of the model in which we borrow all parameter estimates from Smets and Wouters (2007) but allow for inventories, predicts a counterfactually large initial increase in output and inventory-to-sales ratios as well as hours worked. Only by allowing for adjustment costs on inventory investment which increase the firm's shadow valuation of inventories and thus neutralize the effect of real rigidities can the model account simultaneously for the behavior of inventories and factor prices in the data. However in this case, as in the simple Calvo model, the response of the aggregate price level after a monetary shock

is faster than in the Smets and Wouters' model, implying smaller real effects of monetary policy.

Our work is related to a number of recent papers studying the behavior of inventories, costs and markups over the business cycle. Our starting point is the observation by Bils and Kahn that inventories are closely linked to markups and marginal costs and thus may provide important information about the cyclicity of the latter. Khan and Thomas (2007) and Wen (2008) study real business cycle models in which inventories arise due to fixed ordering costs, and a stockout-avoidance motive, respectively. Both of these papers find that the model is capable of accounting for the countercyclicality of the inventory-sales ratio in the data. Our conjecture is that they do so because investment in capital in times of expansions in these models drives up the cost of purchasing (through a higher elasticity of real marginal cost to output) and holding (through higher interest rates) inventories. Most closely related to our analysis is a paper by Jung and Yun (2005) who also study a sticky price model in which firms invest in inventories because these act as a taste shifter in consumer's preferences. They estimate their model by matching impulse responses of aggregate time-series to a monetary shock and find that high rates of depreciation and/or convex costs of deviations from a target inventory-to-sales ratio is necessary to reconcile the model with the data. Finally, Chang, Hornstein and Sarte (2007) study the responses to a productivity shock in a sticky price models with inventories. They find that whether an industry expands or contracts employment depends on the carrying costs of inventories: higher carrying costs prevent firms from responding to the productivity shock by investing in inventories and as a result cut employment given that prices are sticky and quantities demand-determined.

This paper proceeds as follows. In section 2 we briefly review the evidence of the cyclical properties of the inventory-sales ratio and the response of this ratio to identified monetary policy shocks, thus reproducing the facts discussed in Bils and Kahn (2000), Jung and Yun (2005) and in our earlier paper. To build the intuition for our results, in Section 3 we present a simple model of Calvo sticky prices and inventories. Section 4 studies this model's quantitative implications. Section 5 embeds inventories into a richer Smets and Wouters

(2007)-type framework and studies its implications. Section 6 concludes by suggesting several potential resolutions to the challenge of accounting for the behavior of inventories in a model with real rigidities: financing frictions that disconnect fluctuations in the real interest implied by the consumer's pricing kernel from the rate of interest faced by the inventory-carrying firms; additional sources of countercyclical markups; additional frictions that reduce firms' ability to purchase and carry inventories and hence the sensitivity of inventories to costs.

## 2. Evidence

Figure 1 reports the time-series behavior of the inventory-to-sales ratio, as well as output in the annual NBER Manufacturing Productivity Database. The figure shows that the two series are strongly negatively correlated. In fact, as Table 1 shows, the correlation is equal to -0.52 for this annual time-series. The Table also reports moments of the inventory-to-sales ratio for the monthly Manufacturing and Trade Sectors from the Bureau of Economic Analysis National Income and Product Account. The manufacturing and trade sectors jointly account for 85% of the non-farm inventory stocks in the US data, hence our focus on them. The Table shows that the mean inventory-to-sales ratio is 1.41 in the monthly data and 0.23 in the annual NBER Productivity data. These series are highly persistent, but most importantly are strongly negatively correlated with output and sales in these respective sectors at business cycle frequencies.<sup>8</sup> Table 2 reports elasticities of the inventory-to-sales ratio with respect to output and sales. Focusing on the upper row, the Manufacturing and Trade Sector, we find that a 1% increase in output over the cycle lowers the inventory-to-sales ratio by roughly 0.8%. We argue below, in the spirit of *Bils and Kahn (2000)* and *Kahn and Thomas (2007)*, that this elasticity is an important feature of the data against which to evaluate the empirical performance of business cycle models.

Although these correlations do not condition on the source of output fluctuations, we

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<sup>8</sup>We document in *Kryvtsov and Midrigan (2008)* that the inventory-to-sales ratios are negatively correlated with each sector's output and sales even when we focus separately on inventories at different stages of disaggregation (work-in-progress, intermediate goods, and final goods). The reason our results differ from those of *Iachoviello and Schuh (2008)* is that they report correlations with aggregate GDP, not sector-specific GDP.

show in our earlier paper, as do Jung and Yun (2005), that the inventory-to-sales ratio also drops in response to identified expansionary monetary policy shocks, which are our focus in this paper.

### 3. Simple Calvo model with inventories

We start by describing the role of real rigidities in shaping the dynamic response of inventories to a monetary shock in the context of a simple model in which firms are allowed to reprice infrequently, with an exogenous probability  $1 - \xi_p$ , as in the work of Calvo (1983) and Yun (1996). In addition, we assume that firms must choose prices and inventory holdings before an idiosyncratic taste shock is realized. Moreover, the firm is subject to a constraint that it cannot sell more units of the good than its stock of inventories and that it cannot return whatever inventories it has at the end of the period after making the sale (but returns are possible at the beginning of next period). Together these features make it optimal for firms to hold inventories from one period to another. We start by describing the behavior of consumers, producers and the monetary authority.

#### A. Consumers

Consumers have preferences over a continuum of consumption goods and leisure and maximize

$$\begin{aligned} & \max_{c_t(i), n_t, \mathbf{b}_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, n_t) \\ \text{s.t. } & \int_0^1 P_t(i) c_t(i) di + \mathbf{q}_t \cdot \mathbf{b}_{t+1} \leq W_t n_t + b^t + \Pi_t \\ & c_t = \left( \int_0^1 v_t(i)^{\frac{1}{\theta}} c_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \\ & c_t(i) \leq z_t(i) \end{aligned}$$

Here  $\mathbf{b}_{t+1}$  is a vector of state-contingent Arrow-Debreu securities that the consumer

buys and  $\mathbf{q}_t$  is a vector of security prices,  $b^t$  is the quantity of the respective state's bonds the agent has purchased at  $t - 1$ ,  $\Pi_t$  is firm profits,  $W_t$  is the nominal wage rate,  $n_t$  is labor supply,  $c_t(i)$  is consumption of the different varieties, and  $P_t(i)$  their prices. Finally  $c_t$  is the CES aggregator over different varieties and  $v_t(i)$  is a preference shock specific to each good. We assume  $\log(v_t) \sim N(0, \sigma^2)$ . In this economy the consumer will occasionally be turned down by stores with little inventory available for sales. We let  $z_t(i)$  be each firm's available stock of inventories: the consumer cannot buy more than  $z_t(i)$  units.

It is straightforward to show that the consumer's optimal decision rules for  $c_t(i)$ ,  $b_t(s^{t+1})$  and  $n_t$  are:

$$c_t(i) = v_t(i) \left( \frac{P_t(i)}{P_t} \right)^{-\theta} c_t \quad \text{if } v_t(i) \left( \frac{P_t(i)}{P_t} \right)^{-\theta} c_t < z_t(i)$$

$$c_t(i) = z_t(i) \quad \text{otherwise}$$

where  $P_t$  is defined as

$$P_t = \left[ \int_0^1 v_t(i) [P_t(i) + \mu_t(i)]^{1-\theta} di \right]^{\frac{1}{1-\theta}}$$

and  $\mu_t(i)$  is the product of the multipliers on the consumer's budget constraint and on the  $c_t(i) \leq z_t(i)$  constraint. Because some producers stockout in equilibrium  $P_t$  is larger than  $\hat{P}_t = \left[ \int_0^1 v_t(i) P_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}$ , the usual formula for the aggregate price index. Thus financing the same level of the composite consumption good requires a higher expenditure in this setup with love-for-variety and stockouts.

Finally, the optimal consumption-leisure choice satisfies:

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}$$

and the price of a security that pays 1 unit of currency if state  $s^{t+1}$  is realized is:

$$q(s^{t+1}|s^t) = \beta\pi(s^{t+1}|s^t) \frac{U_c(s^{t+1})}{U_c(s^t)} \frac{P(s^t)}{P(s^{t+1})}$$

## B. Firms

Storable final goods are produced by monopolistically competitive firms using a production function linear in labor:

$$y_t(i) = l_t(i)$$

A firm gets to reset its nominal price in any given period with an exogenous probability  $1 - \xi_p$ . Conditional on being allowed to reprice the firm's problem is to choose its nominal price  $P_t(i)$  and inventory stock  $z_t(i)$  that is available for sale at the beginning of the period. As noted above, the price and inventory decisions are made before the firm's idiosyncratic taste shock is realized.

It is convenient to present the firm's problem in recursive form. Drop firm subscripts and let  $\eta$  denote the aggregate state,  $P(\eta), W(\eta), C(\eta)$  denote the aggregate price level, nominal wage, and the composite consumption at state  $\eta$ . Finally, let  $p$  denote the firm's choice of price,  $z$  its choice of how much inventories to have available for sale, and  $s_{-1}$  denote the firm's beginning-of-period inventory holdings. Let  $q(p, z, v; \eta) = \min\left(v \left(\frac{p}{P(\eta)}\right)^{-\theta} c(\eta), z\right)$  denote the firm's sales if it starts with  $z$  units of inventories, charges a price  $p$  and faces a taste shock  $v$ . Then the value of a firm that is allowed to reset its price is:

$$V^a(s_{-1}; \eta) = \max_{p, z} \frac{p}{P(\eta)} \int_v q(p, z, v; \eta) dF(v) - \frac{W(\eta)}{P(\eta)} (z - s_{-1}) + \beta \xi_p \int_v \int_\varepsilon \frac{U_c(\eta')}{U_c(\eta)} V^n(p, s'_{-1}, \eta'(\varepsilon)) dF(v) dG(\varepsilon) + \beta (1 - \xi_p) \int_v \int_\varepsilon \frac{U_c(\eta')}{U_c(\eta)} V^a(s'_{-1}, \eta'(\varepsilon)) dF(v) dG(\varepsilon)$$

where  $\eta'(\varepsilon)$  is the law of motion for the aggregate state (it depends, in particular, on the realization of the aggregate uncertainty,  $\varepsilon$ , here the monetary shock),  $G(\varepsilon)/F(v)$  are the cdfs of aggregate/taste shocks, and  $V^n(p, \cdot, \cdot)$  is the value of the firm that cannot reset its previous price  $p$ . The law of motion for the beginning-of-period inventories is:

$$s'_{-1} = (1 - \delta) [z - q(p, z, v; \eta)]$$

That is, the firm retains the difference between what inventories it made available for sale,  $z$ , and whatever it sells, where we impose the constraint that the firm's sales cannot exceed  $z$ :  $q(p, z, v; \eta) = \min\left(v\left(\frac{p}{P}\right)^{-\theta} c, z\right)$ . Notice here that the per-period dividend payments in the above formulation are the difference between (real) revenues  $\frac{p}{P(\eta)} \int_v q(p, z, v) dF(v)$  and the (real) labor cost associated with replenishing the original stock  $s_{-1}$  to  $z$ .

Similarly, the value of the firm that is unable to reset its price and inherits a nominal price  $p_{-1}$  is:

$$V^n(p_{-1}, s_{-1}; \eta) = \max_z \frac{p_{-1}}{P(\eta)} \int_v q(p_{-1}, z, v; \eta) dF(v) - \frac{W(\eta)}{P(\eta)} (z - s_{-1}) + \beta \xi_p \int_v \int_\varepsilon \frac{U_c(\eta')}{U_c(\eta)} V^n(p_{-1}, s'_{-1}, \eta'(\varepsilon)) dF(v) dG(\varepsilon) + \beta (1 - \xi_p) \int_v \int_\varepsilon \frac{U_c(\eta')}{U_c(\eta)} V^a(s'_{-1}, \eta'(\varepsilon)) dF(v) dG(\varepsilon)$$

with a law of motion for inventories of  $s'_{-1} = (1 - \delta) [z - q(p_{-1}, z, v; \eta)]$ .

Although we find it easier to present the firm's problem in recursive form, the notation is much less cumbersome if we recast the problem in a sequence form. A firm that gets to reset its price at date  $t$  faces the following problem of choosing its nominal price  $p_t$  (recognizing that this price is in effect at date  $t + s$  with probability  $\xi_p^s$  and dropping the dependence on

i) and a sequence of inventory stocks  $z_{t+s}$ ,  $s = 0, \dots, \infty$  in order to solve:

$$\begin{aligned} & \max_{p_t, \{z_{t+s}\}} E_t \sum_{s=0}^{\infty} \xi_p^s Q_{t+s} \left[ p_t \min \left( v_{t+s} \left( \frac{p_t}{P_{t+s}} \right)^{-\theta} c_{t+s}, z_{t+s} \right) - W_{t+s} (z_{t+s} - s_{t+s-1}) \right] + \\ & (1 - \delta) \sum_{s=1}^{\infty} \xi_p^{s-1} (1 - \xi_p) Q_{t+s} W_{t+s} s_{t+s-1} \end{aligned}$$

where

$$s_{t+s} = (1 - \delta) \left[ z_{t+s} - \min \left( v_{t+s} \left( \frac{p_t}{P_{t+s}} \right)^{-\theta} c_{t+s}, z_{t+s} \right) dF(v_{t+s}) \right]$$

are the undepreciated stock of inventories the firm carries over into the next period, and

$$Q_{t+s} = \beta^s \frac{U_{ct+s}}{U_{c,t}} \frac{P_t}{P_{t+s}}$$

Notice that these expectations are over the realization of the demand disturbance at each date  $t + s$ , as well as the aggregate shock, but that the price and inventory decisions are made in each period after the realization of these shocks. We can then write

$$\begin{aligned} R_{t+s}(p_t, z_{t+s}) &= \int_0^1 \min \left( v_{t+s} \left( \frac{p_t}{P_{t+s}} \right)^{-\theta} c_{t+s}, z_{t+s} \right) dF(v_{t+s}) = \\ & P_{t+s}^\theta c_{t+s} \exp \left( \frac{\sigma^2}{2} \right) F \left( \log \left( \frac{z_{t+s}}{\left( \frac{p_t}{P_{t+s}} \right)^{-\theta} c_{t+s}} \right) - \sigma^2 \right) + z_{t+s} \left( 1 - F \left( \log \left( \frac{z_{t+s}}{\left( \frac{p_t}{P_{t+s}} \right)^{-\theta} c_{t+s}} \right) \right) \right) \end{aligned}$$

The problem can then be rewritten as:

$$\begin{aligned} & \max_{p_t, \{z_{t+s}\}} E_t \sum_{s=0}^{\infty} \xi_p^s Q_{t+s} [p_t R_{t+s}(p_t, z_{t+s}) - W_{t+s} (z_{t+s} - (1 - \delta) [z_{t+s-1} - R_{t+s-1}(p_t, z_{t+s-1})])] \\ & + (1 - \delta) \sum_{s=1}^{\infty} \xi_p^{s-1} (1 - \xi_p) Q_{t+s} W_{t+s} [z_{t+s-1} - R_{t+s-1}(p_t, z_{t+s-1})] \end{aligned}$$



Notice here that the firm's objective can be rewritten more transparently as:

$$W_t s_{t-1} + E_t \left( \begin{aligned} & [(p_t - (1 - \delta) Q_{t+1} W_{t+1}) R_t(p_t, z_t) - (W_t - (1 - \delta) Q_{t+1} W_{t+1}) z_t] + \\ & \xi_p Q_{t+1} \left[ \left( p_t - (1 - \delta) \frac{Q_{t+2}}{Q_{t+1}} W_{t+2} \right) R_{t+1}(p_t, z_{t+1}) - \left( W_{t+1} - (1 - \delta) \frac{Q_{t+2}}{Q_{t+1}} W_{t+2} \right) z_{t+1} \right] + \\ & \dots \\ & \xi_p^s Q_{t+s} \left[ \left( p_t - (1 - \delta) \frac{Q_{t+s+1}}{Q_{t+s}} W_{t+s+1} \right) R_{t+s}(p_t, z_{t+s}) - \left( W_{t+s} - (1 - \delta) \frac{Q_{t+s+1}}{Q_{t+s}} W_{t+s+1} \right) z_{t+s} \right] + \dots \end{aligned} \right)$$

This expression has a straightforward interpretation. The firm's expected sales, given its price  $p_t$  and inventory stock  $z_{t+s}$  are equal to  $R_{t+s}(p_t, z_{t+s})$ . Each unit sold generates profits equal to  $p_t - (1 - \delta) \frac{Q_{t+s+1}}{Q_{t+s}} W_{t+s+1}$ . The latter term is the marginal valuation of inventories the firm sells as this is how much a unit of inventories is worth to the firm given the irreversibility (no returns after committing to a given level of  $z_{t+s}$  are allowed contemporaneously): the firm takes into account the fact that the good depreciates, and can be sold at  $W_{t+s+1}$  (and is thus worth  $(1 - \delta) \frac{Q_{t+s+1}}{Q_{t+s}} W_{t+s+1}$  at date  $t + s$ ).

Thus, absent the sticky price friction, the firm would choose its price as a markup (which depends on the elasticity of  $R$  with respect to price) over  $(1 - \delta) \frac{Q_{t+s+1}}{Q_{t+s}} W_{t+s+1}$ . Given the price setting friction, the optimal price is an appropriately weighted average of these shadow valuations, as will be shown below.

Consider next the problem of how much inventories the firm should make available for sale,  $z_{t+s}$ . The gain from holding inventories is that it relaxes the stockout constraint. The cost is given by  $\left( W_{t+s} - (1 - \delta) \frac{Q_{t+s+1}}{Q_{t+s}} W_{t+s+1} \right) z_{t+s}$ : an additional unit of (unsold inventories) costs the firm  $W_{t+s}$  at date  $t + s$  and is only worth  $(1 - \delta) \frac{Q_{t+s+1}}{Q_{t+s}} W_{t+s+1}$  next period.

The first order conditions are thus:

$$[z_{t+s}] : 1 - F(v_{t+s}^*) = \frac{W_{t+s} - (1 - \delta) E_{t+s} \frac{Q_{t+s+1}}{Q_{t+s}} W_{t+s+1}}{p_t - (1 - \delta) E_{t+s} \frac{Q_{t+s+1}}{Q_{t+s}} W_{t+s+1}} \quad (1)$$

$$[p_t] : E_t \sum_{s=0}^{\infty} \xi_p^s Q_{t+s} \left[ R_{t+s}(p_t, z_{t+s}) + \left( p_t - (1-\delta) \frac{Q_{t+s+1}}{Q_{t+s}} W_{t+s+1} \right) R_{p,t+s}(p_t, z_{t+s}) \right] = 0 \quad (2)$$

where  $v_{t+s}^* = \left[ \left( \frac{p_t}{P_t} \right)^{-\theta} c_t \right]^{-1}$   $z_t$  is the cutoff taste shock at which a firm with price  $p_t$  and inventories  $z_t$  does not stockout. Equation (1) gives the optimal choice of the (presale) stock-sale ratio. The left-hand side is the probability of stocking out which decreases with the inventory-to-(median) sale ratio. The marginal cost of carrying an additional unit of inventories,  $W_{t+s} - (1-\delta)E_{t+s}Q_{t+s}W_{t+s+1}$ , must equal to the marginal benefit: the profits made from an additional unit sold,  $p_t - (1-\delta)E_{t+s}Q_{t+s}W_{t+s+1}$ , multiplied by the probability of stockout,  $1 - F(v_{t+s}^*)$ . Equation (2) is the pricing equation which says that the firm sets its price  $p_t$  so that the present discounted value of marginal revenue is equal to the present discounted value of the shadow valuation of inventories, given by subsequent period's replacement value.

### C. Equilibrium

We next derive the aggregate price index in this economy. Recall from the above that this is given by:

$$P_t = \left[ \int_0^1 v_t(i) [P_t(i) + \mu_t(i)]^{1-\theta} di \right]^{\frac{1}{1-\theta}}$$

As above, let  $v_t^*(i) = \left[ \left( \frac{P_t(i)}{P_t} \right)^{-\theta} c_t \right]^{-1}$   $z_t(i)$  be the maximum level of the taste shock for which a firm with inventories  $z_t(i)$  does not stock out, i.e., for which its current inventories are sufficient to meet demand. We can then use the fact that  $z_t(i) = v_t^*(i) \left( \frac{P_t(i)}{P_t} \right)^{-\theta} c_t = v_t(i) \left( \frac{P_t(i) + \mu_t(i)}{P_t} \right)^{-\theta} c_t$  to write:

$$[P_t(i) + \mu_t(i)]^{1-\theta} = \left( \frac{v_t^*(i)}{v_t(i)} \right)^{\frac{\theta-1}{\theta}} P_t(i)^{1-\theta}$$

which gives the following expression for the aggregate price level:

$$\begin{aligned} P_t^{1-\theta} &= \left[ \int_{v_t(i) \leq v_t^*(i)} v_t(i) P_t(i)^{1-\theta} di + \int_{v_t(i) > v_t^*(i)} v_t(i) \left( \frac{v_t^*(i)}{v_t(i)} \right)^{\frac{\theta-1}{\theta}} P_t(i)^{1-\theta} di \right] \\ &= \int_0^1 P_t^{1-\theta}(i) \Phi(v_t^*(i)) di \end{aligned}$$

where we invoke the law of large numbers and the fact that  $v_t(i)$  is independent of  $P_t(i)$  and  $z_t(i)$ . Here  $\Phi(v_t^*(i))$  satisfies:

$$\Phi(v_t^*(i)) = \int_{v_t(i) \leq v_t^*(i)} v_t(i) dF(v) + v_t^*(i)^{\frac{\theta-1}{\theta}} \int_{v_t(i) > v_t^*(i)} v_t(i)^{\frac{1}{\theta}} dF(v)$$

For normally distributed demand shocks

$$\Phi(v_t^*(i)) = \exp\left(\frac{\sigma^2}{2}\right) F\left(\log v_t^*(i) - \sigma^2\right) + v_t^*(i)^{\frac{\theta-1}{\theta}} \exp\left(\frac{\sigma^2}{2\theta^2}\right) \left(1 - F\left(\log v_t^*(i) - \frac{1}{\theta}\sigma^2\right)\right)$$

Finally, we assume a money demand function of the form:

$$M_t = P_t c_t$$

We solve the model by log-linearizing the problem around a steady-state with a constant money supply. Appendix 1 presents detailed derivations.

## 4. Parameterization and Experiments

Table 3 presents the parameter values we use in our numerical work. The period is a month. The discount factor corresponds to an annual interest rate of 4%. We assume that the frequency of price changes is 0.18, corresponding to 5 months of average price duration. The elasticity of substitution,  $\theta$ , is set equal to 5, a number in the range of those used in

earlier work. We assume a depreciation rate is 0.3% corresponding to steady state ratio of inventory investment to output of 0.004 as in NIPA data. Finally, the standard deviation of demand shocks is chosen so as to match an end-of-period inventory-to-sales ratio of 1.4 as in the NIPA data. Average frequency of stockouts predicted by the model is 2.3%. Finally, we assume a utility function  $u(c, n) = \log(c) - \psi n$ , implying a consumption-leisure choice of

$$\frac{W}{P} = \psi c$$

Given that  $M = Pc$ , we have that in this benchmark calibration the nominal wage is proportional to the money stock and that the elasticity of real marginal cost to consumption is 1.

In all experiments we consider the impulse response in this economy to an unanticipated one-time 1% increase in the stock of money  $M$ . We refer to the benchmark economy presented above in which the elasticity of real marginal cost to consumption is equal to unity as one with "No Real Rigidities".

To model real rigidities, we resort to the following reduced-form approach. We modify consumer preferences to  $u(c_t, n_t) = \log(c_t) - \psi_t n_t$  where  $\psi_t$  is a time-varying preference shock (wedge) that is assumed to be correlated with the monetary shock in such a way so as to lower the elasticity of real wage to consumption below unity. In particular, we assume that  $\psi_t$  is such that the path of the nominal wage is equal to

$$\log(W_t) = \xi_w \log(W_{t-1}) + (1 - \xi_w) \log(\psi P c)$$

Here  $\xi_w$  determines the speed with which the nominal wage converges to its steady-state value. In the benchmark model  $\xi_w = 0$  and the wage adjusts immediately in response to the monetary disturbance. By appropriately choosing the path for  $\psi_t$  we can increase  $\xi_w$  in our experiments so as to capture greater degrees of real rigidities (here arising from sticky wages). We think of this approach as a reduced-form approach to modeling frictions in the

labor and intermediate goods markets, as well as assumptions on technology and preferences used in earlier applied work in order to slow down the response of the real marginal cost of production to a monetary disturbance. In the next section we model these wedges explicitly.

### A. No Real Rigidities

Figure 2 plots impulse responses to a one-time 1% increase in the money stock in our benchmark economy with a unitary elasticity of real marginal cost to output. The bottom right panel shows the responses of inventory investment (Inv, in % of output), sales (S), and inventory-to-sales ratio (IS), the latter as % deviations from their steady-state levels. Clearly, the inventory-to-sales ratio drops after the shock, as in the data. To see what accounts for this drop, notice that the optimal inventory choice can be rewritten as:

$$1 - F(v_t^*) = \frac{1 - (1 - \delta)\beta E_t \frac{U_{c,t+1}}{U_{c,t}} \frac{W_{t+1}/P_{t+1}}{W_t/P_t}}{\frac{p_t}{W_t} - (1 - \delta)\beta E_t \frac{U_{c,t+1}}{U_{c,t}} \frac{W_{t+1}/P_{t+1}}{W_t/P_t}}$$

where recall that  $v_t^* = \left[ \left( \frac{p_t}{P_t} \right)^{-\theta} c_t \right]^{-1} z_t$  is the beginning of period's inventory stock relative to the demand for the firm's good if the taste shock were equal to 1. This expression says that the optimal inventory-to-demand ratio increases with the markup  $\frac{p_t}{W_t}$ , as with higher markups stockouts are costlier. Moreover, this ratio also increases with the expected growth of the real wage rate  $\frac{W_{t+1}/P_{t+1}}{W_t/P_t}$  and decreases with the real interest rate rate,  $\beta \frac{U_{c,t+1}}{U_{c,t}}$ . The intuition for why the inventory-to-sales ratio decreases is then straightforward. After a monetary shock real wages are expected to decrease (as prices increase: see upper-left panel of Figure 2). Given our assumption on preferences, the increase in real wages is proportional to  $\frac{1}{U_{c,t+1}}$ : as a result  $\frac{U_{c,t+1}}{U_{c,t}} \frac{W_{t+1}/P_{t+1}}{W_t/P_t}$ , the term capturing the cost of acquiring and holding inventories is unchanged. Thus, given the lower average markups induced by the immediate adjustment of wages and stickiness in prices, firms find it optimal to hold less inventories.

In Table 4 we report the average deviation of the real wage  $\frac{W}{P}$  from its steady state level, relative to the deviation of consumption,  $C$ , from its steady-state value, in the first 5 periods after the shock, as well as a similar measure for the elasticity of inventory-to-sales

to sales. The drop of inventory-to-sales ratio is 0.6 times the increase in sales (so that the elasticity of inventory-sales ratio to sales is -0.6). This elasticity is close to that in the data (-0.8).

## B. Large Real Rigidities

We next assume that nominal wages adjust slowly to the monetary shock. In particular, we assume that  $\xi_w = \frac{1}{2}$ . Figure 3 reports the results of this exercise. When wages adjust slowly to the monetary expansion the real cost of acquiring and holding inventories,  $\frac{U_{c,t+1} W_{t+1}/P_{t+1}}{U_{c,t} W_t/P_t}$ , decreases as the wedge in  $U_{c,t} \frac{W_t}{P_t} = \psi_t$  is negative. In addition, the decrease in markups associated with sticky prices is smaller. Given that our parametrization implies that inventories are very sensitive to costs (the elasticity of inventory-to-sales ratio to expected cost change is 28), and much less so to markups (the elasticity is 0.08), the decrease in the cost of holding inventories dominates and the inventory-to-sales ratio increases. Table 4 summarizes this discussion and shows that the elasticity of real wages to consumption is 0.69 in this experiment, while that of inventory-to-sales to sales is 2.62, much higher than in the data. Notice also in the lower-left panel of Figure 3 that the model's counterfactual inventory responses also generate counterfactually high output responses.

As illustrated by House (2007) and Jung and Yun (2005), the sensitivity of investment in durable goods to costs depends critically on the rate at which these goods depreciate. Low rates of depreciation make it optimal for firms to buy when costs are low as this is when the cost of carrying this additional stock of goods across periods is low. In fact, as Jung and Yun (2005) show, in the context of a model in which inventories enter preferences directly in the definition of the composite consumption good and in which partial indexation of prices to inflation and use of intermediate inputs as a function of production are the source of real rigidities, that allowing for high rates of depreciation (in excess of 50% per quarter) brings the model's impulse responses of inventories to a monetary shock in line with those in the data. Our results above suggest that this sensitivity of inventories to aggregate shocks in the presence of low rates of depreciation is not specific to the specific assumption that makes it

optimal for firms in the model to hold inventories.

### **C. Higher Depreciation Rates and Large Real Rigidities**

In this subsection we ask whether higher rates of depreciation can indeed bring the model's aggregate implications in line with those in the data when the economy is characterized by real rigidities. In Figure 4 we report impulse responses of our model to a monetary shock for several different values of the depreciation rate. Table 5 reports the sales elasticities of inventory-to-sales ratios for different depreciation rates: even with depreciation rates as large as 8% per month we are not able to obtain a decrease in the (end-of-period) inventory-to-sales ratio. For  $\delta = 0.08$  inventories respond one-to-one with sales so that inventory-to-sales ratio does not move after the shock.

Although high rates of depreciation somewhat resolve the model's ability to better reproduce the inventory-sales ratio's second moments, they do so by requiring implausibly large taste shocks (that lead to implausibly large fluctuations in firm-level quantities and prices) to match the first moment of this time-series. In order for firms to be willing to hold the 1.4 end-of-period monthly inventory-to-sales ratio observed in the US data, the model requires a very large standard deviation of the taste shocks ( $\sigma = 237\%$ ). We conclude that high depreciation rates are not a plausible fix of the model's counterfactual implications.

#### ***Adjustment Costs and Large Real Rigidities***

We show here that adding adjustment costs does help resolve the model's counterfactual implications of the model. The intuition is simple: with adjustment costs the firm's cost of purchasing inventories are not only the labor cost (which goes down in relative terms because of the sticky wages) but also the cost of expanding production. Thus the reason inventories do not respond to the monetary expansion is because the marginal cost of acquiring them (and therefore their marginal valuation) is high. In particular, this implies then that the sticky wages do not translate into a slower response of prices at the aggregate level

as firms incorporate the effect of higher adjustment costs by raising prices. In other words adding adjustment costs simply lowers the strength of real rigidities and brings them close to those in the Benchmark economy in which wages adjust fully to the monetary shock.

We assume next quadratic costs of adjusting the stock of inventories<sup>9</sup>. In particular, we modify the inventory accumulation equation from:

$$z_t(i) = s_{t-1}(i) + y_t(i)$$

to one of the following specifications:

$$z_t(i) = s_{t-1}(i) + y_t(i) - \frac{\eta}{2}(y_t(i) - y^*)^2$$

where  $y^*$  is the steady-state output level, or

$$z_t(i) = s_{t-1}(i) + y_t(i) - \frac{\eta}{2}(y_t(i) - y_{t-1}(i))^2$$

Thus in one case we penalize deviations of the firm's output from a given target, and in another we penalize changes in the firm's output. Notice that this modification replaces the firm's inventory accumulation and pricing decisions. In particular, the firm's shadow cost of inventories is now (in log-deviations from the steady state):

$$\begin{aligned}\widehat{\lambda}_t &= \frac{\widehat{W}_t}{P_t} + \eta \widehat{y}_t \\ \widehat{\lambda}_t &= \frac{\widehat{W}_t}{P_t} + \eta [\widehat{y}_t - \widehat{y}_{t+s-1}]\end{aligned}$$

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<sup>9</sup>Jung and Yun (2005) consider an alternative adjustment cost specification that penalizes deviations of a firm's inventory-sales ratio from a given target. We argue in Kryvtsov and Midrigan (2008) that this alternative specification lowers the variability of the aggregate inventory-to-sales ratio by preventing fluctuations in the inventory-to-sales ratio at the firm level. In contrast, a dataset of prices and inventories in Spanish supermarkets studied by Aguirregabiria (1999) shows substantial volatility in the inventory-to-sales ratio at the firm level.



depending on the nature of the adjustment cost. Thus even though the real wage is slow to respond to the monetary shock, the firm's true cost of acquiring inventories increases because of the quadratic adjustment costs.

Figures 5 and 6 report impulse responses to a monetary shock in the presence of adjustment costs of various degrees (Tables 6 and 7 report the implied elasticities). Clearly, adjustment costs slow down the response of output and therefore that of the inventory-to-sales ratio: in fact the model can easily generate impulse responses similar to those in the data. Notice however that the response of inflation is much stronger when we add adjustment costs because of the increase in the marginal valuation of inventories. To see the difference between the Benchmark economy without real rigidities and the economy with sticky wages and adjustment costs, we perform the following experiment in the model where the cost of adjustment is incurred for deviation of output from its steady-state level. We calibrate the size of the adjustment cost,  $\eta$ , in order to match the average elasticity of the inventory-to-sales ratio to output of  $-0.60$  in the benchmark economy without real rigidities. Figure 7 shows that in this economy  $\hat{\lambda}_t$ , the marginal cost of increasing the stock of inventories has an almost identical response as the nominal wage  $W_t$  in the model without sticky wages. The response of prices is thus the same as in the benchmark model as the stickiness in wages is offset by the increase in the adjustment cost.

#### **D. Imposing responses of real interest rate and real wages from the data**

As shown above, the response of the inventory-to-sales ratio is determined by the dynamics of the marginal cost of acquiring additional stock,  $W_t$ , or  $\lambda_t$  in the model with adjustment costs, and the real interest rate (determined in the model by the expected consumption growth). In this subsection, instead of simulating the dynamics of these two variables implied by the model, we impose them directly from the data. Specifically, we assume a process for the growth rate of the money supply and feed the consumption-leisure choice wedges that are chosen so that the model *exactly* reproduces the impulse responses of real wages and real interest rate in the data.

Figure 8 reports impulse responses in the model. Real interest rate decreases on impact thus lowering the inventory-carrying cost. In the data the real wage also responds little to the monetary shock. Thus the data is characterized by even stronger degrees of wage stickiness than what we have imposed in our "Large Real Rigidities" experiments. As a result the inventory-to-sales ratio is procyclical, with an elasticity with respect to sales of 7.89, as shown in Table 8.

## 5. Smets and Wouters (2007) with inventories

We show next that our results are robust to introducing additional features that have received attention in earlier work. In particular we consider a version of the economy studied by Smets and Wouters (2007) to which we add inventories in a similar fashion as we did in the previous section. The model differs from that in the earlier section in that capital is a factor of production (whose rate of utilization can be varied) subject to adjustment costs, wages are sticky because of Calvo-type frictions in a (monopolistically competitive) labor market, as well as the assumption of external habit formation, and a Taylor-type interest rate rule with smoothing. To conserve on space, we present a detailed description of the model in Appendix 2.

We solve the original Smets-Wouters (2007) economy as well as the economy in which we introduce inventories and demand uncertainty. We use the parameter values estimated by Smets and Wouters (2007) and in addition assume a depreciation rate of  $\delta = 0.4\%$  to match the average share of inventory investment to output as well as a standard deviation of idiosyncratic taste shocks to match the inventory-to-sales ratio of 1.4 in the data<sup>10</sup>.

Figure 9 reports impulse responses to a negative s.d. decrease in the interest rate (monetary expansion). The figure compares the responses in the Data, original Smets and Wouters (2007) setup, as well as in the model that adds inventories. Clearly, the responses of

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<sup>10</sup>We have also attempted to re-estimate the Smets and Wouters (2007) economy by adding an additional time-series (inventory-to-sales ratio) to the 7 time series in their original paper. We were unable to re-estimate the model, presumably because of the tension between accounting for the time-series of real wage and inflation series simultaneously with the inventory series.

the model change significantly when we add inventories, for exactly the same intertemporal substitution motive as in the previous section. In particular, the change in inventories is much larger than the response in the data, as is the response of hours and output (for which the maximal impact is instantaneous, despite the habit and capital adjustment frictions), as well as consumption. Notice that the behavior of inflation is pretty much unaffected.

Figure 10 conducts a robustness exercise by varying the rate at which inventories depreciate in the model. Interestingly, even with very high rate of depreciation (40% per month), the model is unable to reproduce the countercyclical inventory-to-sales response in the data, although clearly the model's predictions for aggregate variables improve (while those for the volatility of the firm-level quantities worsen because much larger taste shocks are necessary to induce firms to hold the 1.4 ratio of inventories-to-sales in the data).

Figure 11 varies the size of adjustment costs in inventory accumulation (modeled here as penalizing changes in output): as above, sufficiently high adjustment costs slow down the response of inventories, and therefore can produce countercyclical inventory-to-sales ratios.

Finally, we re-estimated the Smets and Wouters (2007) economy by allowing quadratic adjustment costs and adding the inventory-to-sales ratio as an additional time-series. The best fit is offered by a model with a size of adjustment costs equal to  $\eta = 1.67$ . Figure 12 presents the results. In this case the inventory-to-sales ratio is acyclical (elasticity of -0.01 vs. -0.8 in the data). Moreover, the response of inflation is much quicker than in the original Smets-Wouters (2007) setup. As above, adding adjustment costs allows the model to simultaneously match the behavior of the real wage series and inventories in the data, by increasing the cost of inventory accumulation, and therefore the shadow valuation and price of inventories. Quicker price responses imply at the aggregate level shorter-lived real effects of monetary disturbances relative to those in the model with real rigidities.

## 6. Conclusions

In this paper we have considered a number of extensions to our analysis in Kryvtsov and Midrigan (2008) in which we study the behavior of inventories in a model with sticky prices and real rigidities. We find that, consistent with our earlier results and the results in Jung and Yun (2005), inventories are highly sensitive to cyclical fluctuations in the cost of acquiring and holding them. This is true both in a simple Calvo model with a stockout-avoidance motive for holding inventories, as well as in a richer Smets-Wouters (2007) - type model. Thus even small amounts of real rigidities predict a counterfactually high increase in the inventory-to-sales ratio in response to monetary expansions. In contrast, in the data this ratio persistently declines in times of booms. We have shown that adding adjustment costs on output (or more generally factors of production) allows the model to simultaneously match the behavior of real wages and other time-series in the data. This modification implies, however, that a firm's shadow valuation of inventories increases sharply during booms despite the sluggishness of factor prices. As a result, the implications for inflation behavior in the model that is capable of accounting for the observed behavior of inventories, resemble those in a model with little real rigidities.

We conclude that standard models of inventories pose a challenge for New Keynesian sticky price models in which real rigidities take the form of slow responsiveness of real marginal cost to output (e.g. sticky wages or intermediate good's prices, variable factor utilization etc.). Potential resolutions to this challenge include a) allowing for financing frictions that disconnect fluctuations in the real interest implied by the consumer's pricing kernel from the rate of interest faced by inventory-carrying firms, b) allowing additional sources of countercyclical markups (other than nominal price rigidities) that would decrease the benefits of carrying inventories during booms and c) additional frictions on the firms' ability to purchase and carry inventories (e.g., non-linear rates of depreciation, capacity constraints) that reduce the sensitivity of inventories to costs. Exploring these alternatives is an interesting avenue for future research.

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Table 1: Inventory-to-Sales Ratio in US Data

	Mean	Std	Serr corr	Correlation with	
				output	sales
<i>NIPA monthly</i>					
Manufacturing and Trade	1.41	2.19	0.88	-0.83	-0.83
Retail	1.31	2.08	0.72	-0.49	-0.61
<i>NBER annual</i>					
Manufacturing	0.31	3.90	0.31	-0.52	-0.66

Note: This Table reproduces Table 1 in Kryvtsov and Midrigan (2008) . Data are from the BEA National Income and Product Accounts (monthly) from January 1967 to December 1997 and the NBER Manufacturing Productivity Database from 1957 to 1996. All data are HP filtered. Output, sales and inventory-to-sales ratio are defined in % deviations from respective HP trends. Inventory investment is defined in % points-of-output-fraction.



Table 2: Elasticity of Inventory-to-Sales Ratio

	output elasticity	sales elasticity
<i>NIPA monthly</i>		
Manufacturing and Trade	-0.77	-0.86
Retail	-0.49	-0.70
<i>NBER annual</i>		
Manufacturing	-0.42	-0.60

Note: This Table reproduces Table 3 in Kryvtsov and Midrigan (2008) . Data are taken from BEA National Income and Product Accounts monthly data from January 1967 to December 1997 and the NBER Manufacturing Productivity Database from 1957 to 1996. Elasticities are regression coefficients with log inventory-to-sales as dependent variable and log output (or log sales) as independent variable, in addition to fixed time effects.

Table 3: Parameter values.

Parameter		Value
discount factor	$\beta$	$0.96^{1/12}$
degree of nominal price stickiness	$\xi_p$	0.82
elasticity of goods demand	$\theta$	5
depreciation rate of stock, %	$\delta$	0.3
standard deviation of demand shocks, %	$\sigma_v$	51.8
<i>Additional moments</i>		
frequency of stockouts, %		2.3

Table 4: Elasticities of real wages and I/S to sales.

	Real rigidity	mean $\frac{\widehat{I/S}}{\widehat{S}}$	mean $\frac{\widehat{W/P}}{\widehat{C}}$
Benchmark	$\lambda = 0$	-0.60	1
Large real rigidities	$\lambda = \frac{1}{2}$	2.62	0.69

Table 5: Elasticities in Calvo model with stockouts:  
high depreciation rates.

	$\frac{\widehat{I/S}}{\widehat{S}}$	$\frac{\widehat{W/P}}{\widehat{C}}$
$\delta = 0.0029$	2.62	0.69
$\delta = 0.025$	0.51	0.66
$\delta = 0.05$	0.16	0.62
$\delta = 0.065$	0.02	0.58

Table 6: Elasticities in Calvo model with stockouts:  
adjustment cost of deviating from steady-state level.

	$\frac{\widehat{I/S}}{\widehat{S}}$	$\frac{\widehat{W/P}}{\widehat{C}}$
$\eta = 0$	2.62	0.69
$\eta = 0.2$	0.22	0.66
$\eta = 0.5$	-0.71	0.63
$\eta = 1$	-1.43	0.60

Table 7: Elasticities in Calvo model with stockouts:  
adjustment cost on output changes.

	$\frac{\widehat{I/S}}{\widehat{S}}$	$\frac{\widehat{W/P}}{\widehat{C}}$
$\eta = 0$	2.62	0.69
$\eta = 0.3$	0.19	0.68
$\eta = 0.7$	-0.76	0.67
$\eta = 1$	-1.13	0.67

Table 8: Imposing IRFs for real wages and real interest rate

Imposed IRFs	mean $\frac{\widehat{I/S}}{\widehat{S}}$	mean $\frac{\widehat{W/P}}{\widehat{C}}$
real wage, real interest rate	7.89	0.29

Table 9: Parameter estimates in various Smets-Wouters (2007) economies

		SW	SWI	SW-adj
habit formation	$\lambda$	0.71	0.71	0.59
elasticity of capital adj. cost	$\varphi$	5.2	5.2	4.1
elasticity of cap. utilization adj. cost	$\psi$	0.59	0.59	0.41
degree of nominal price stickiness	$\xi_p$	0.81	0.81	0.81
price indexation to lagged inflation	$\iota_p$	0.24	0.24	0.09
curvature of Kimball goods market aggregator	$\varepsilon_p$	0	0	0
degree of nominal wage stickiness	$\xi_w$	0.81	0.81	0.88
wage indexation to lagged inflation	$\iota_w$	0.64	0.64	0.54
curvature of Kimball labor market aggregator	$\varepsilon_w$	0	0	0
inventory adj. cost	$\eta$	–	0	1.67

Note: "SW" - original Smets and Wouters (2007) model (estimated with MLE without kinked demand in labor and goods, "SWI" - "SW" model with inventories (not reestimated), "SW-adj" - SW model with inventories and adjustment cost (reestimated).

Figure 1: Aggregate Inventory-to-Sales Ratio, NBER Database

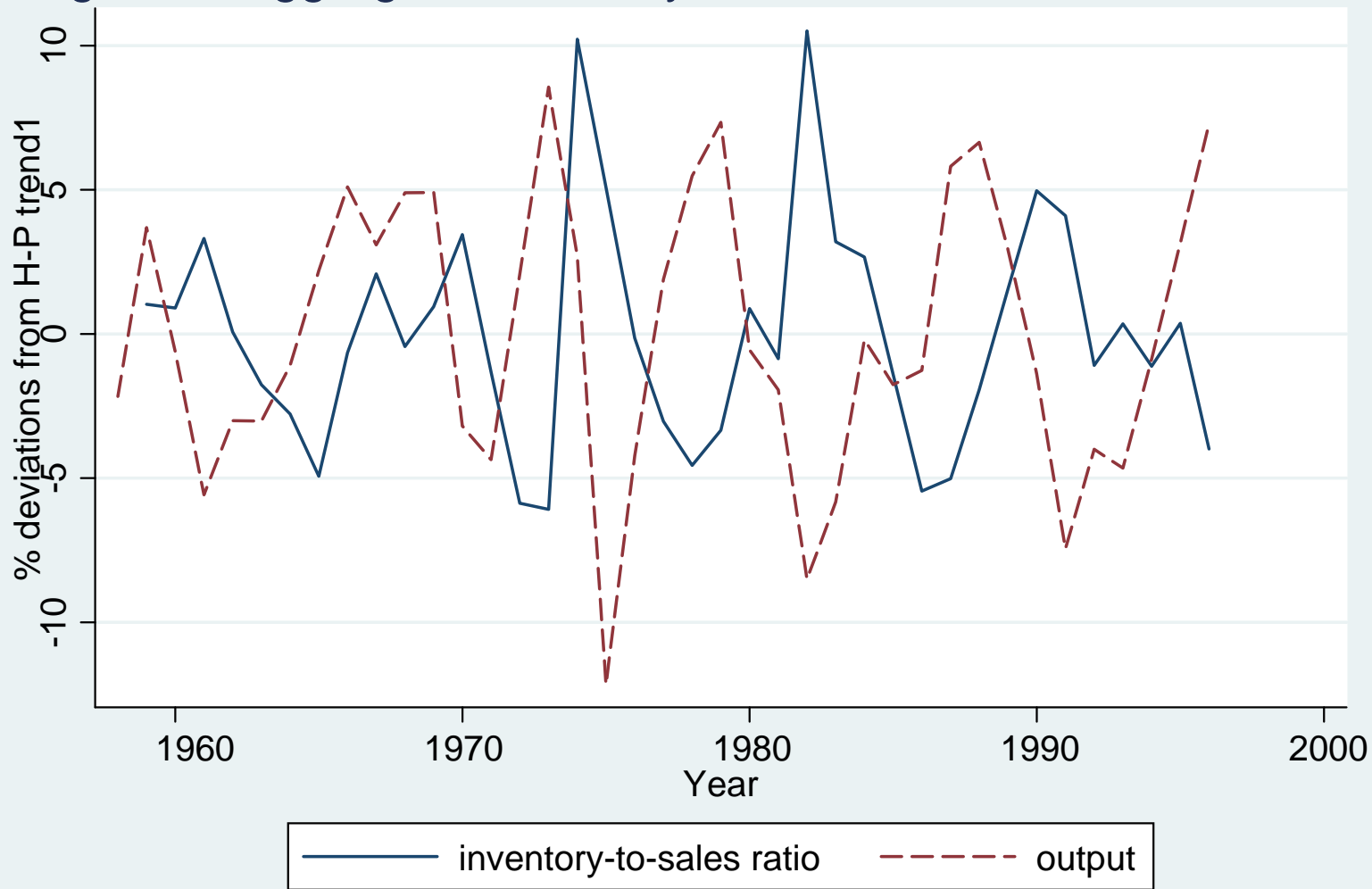


Figure 2: Simple Calvo, no real rigidities

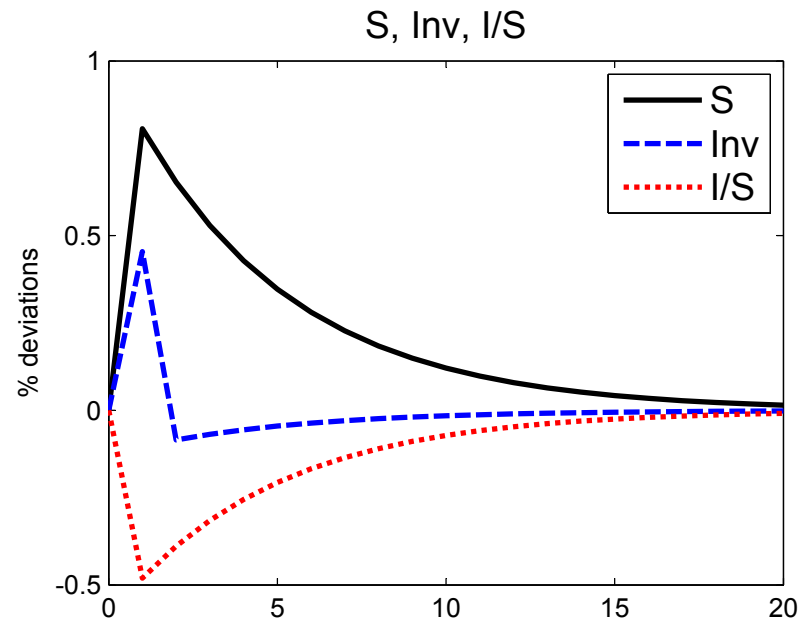
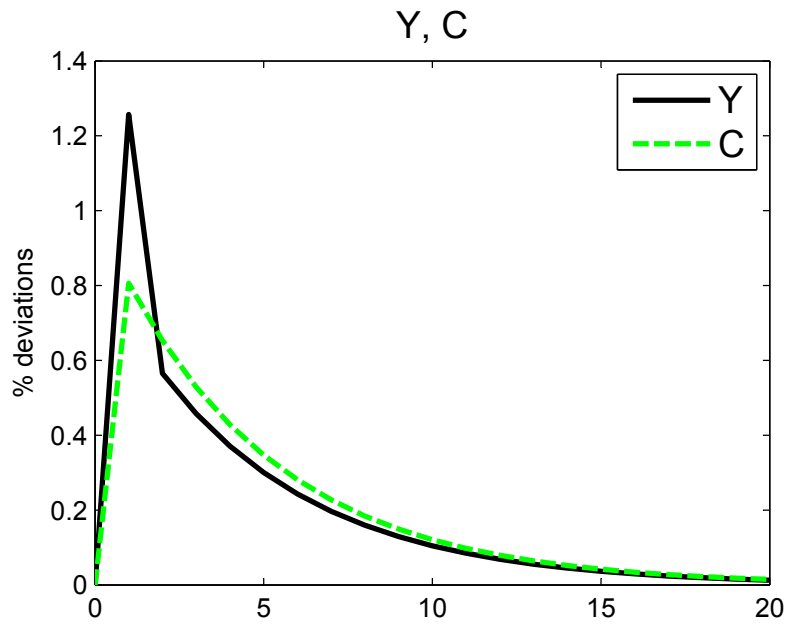
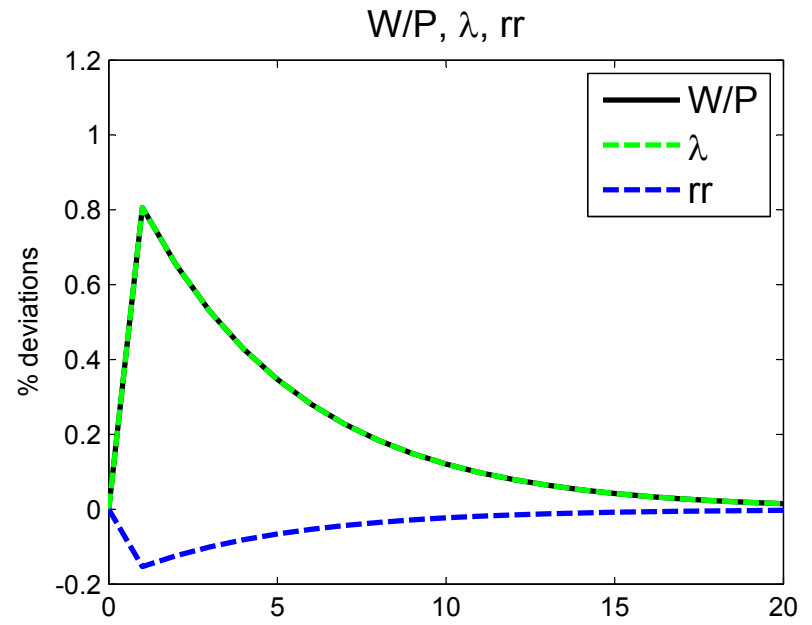
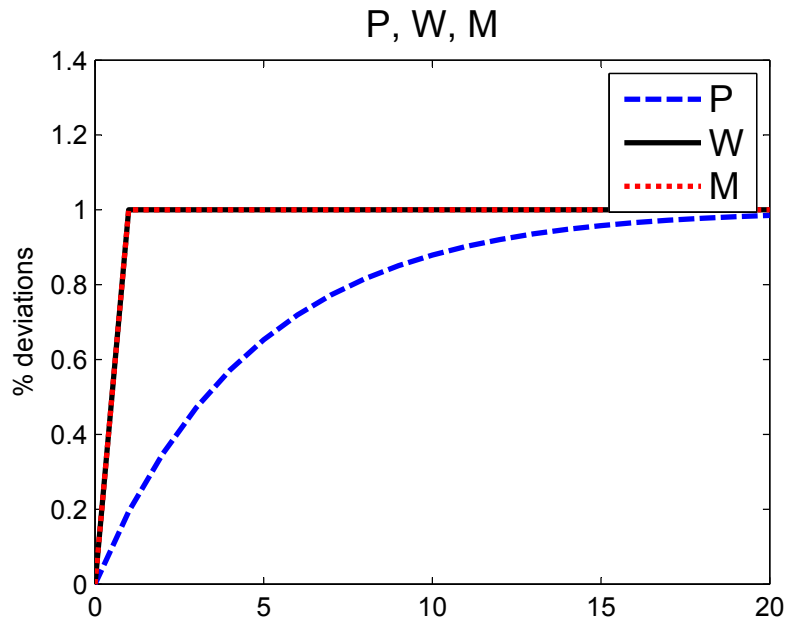


Figure 3: Simple Calvo, large real rigidities

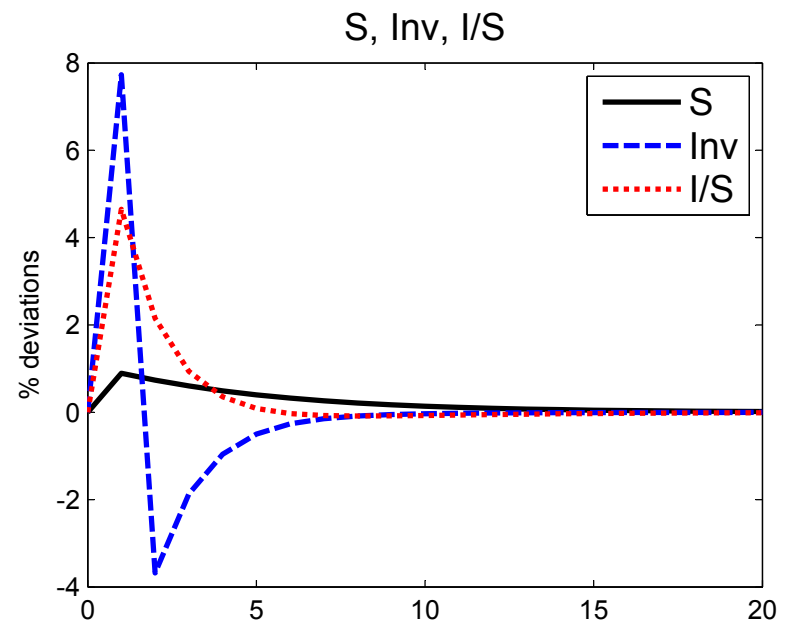
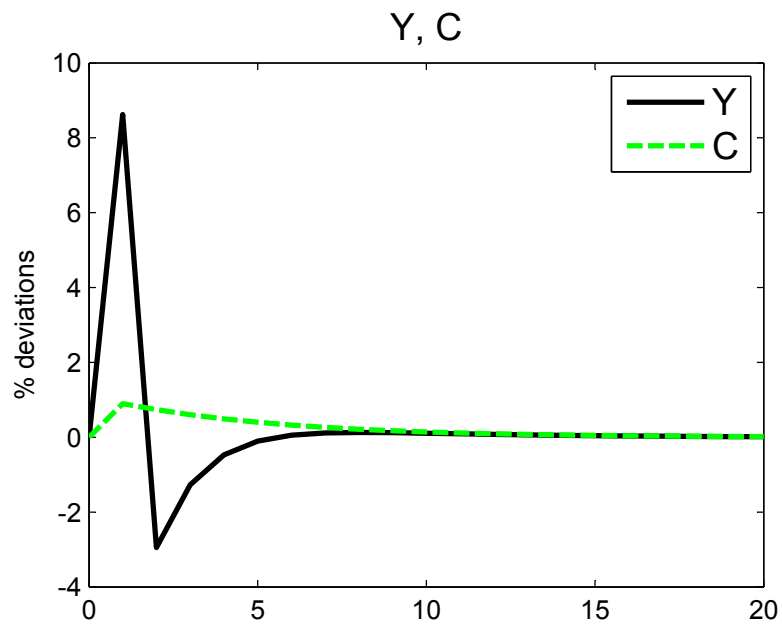
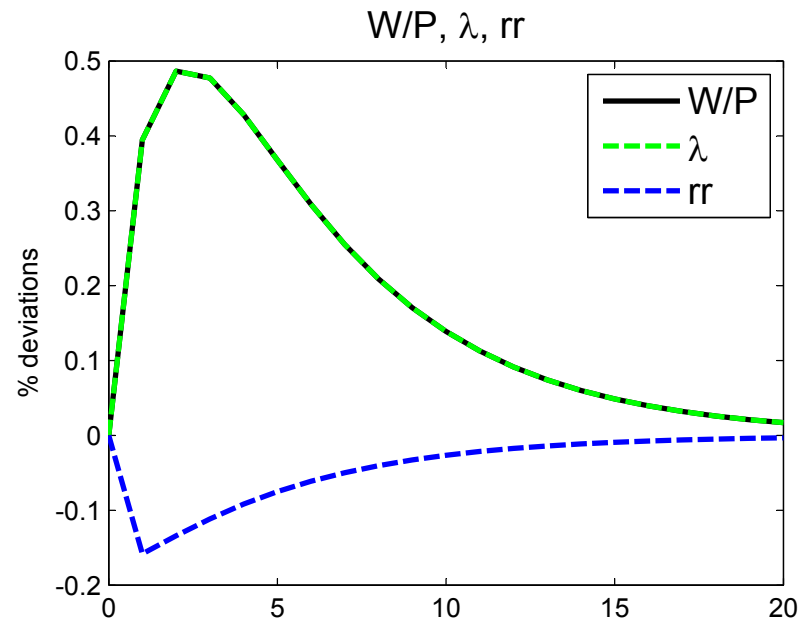
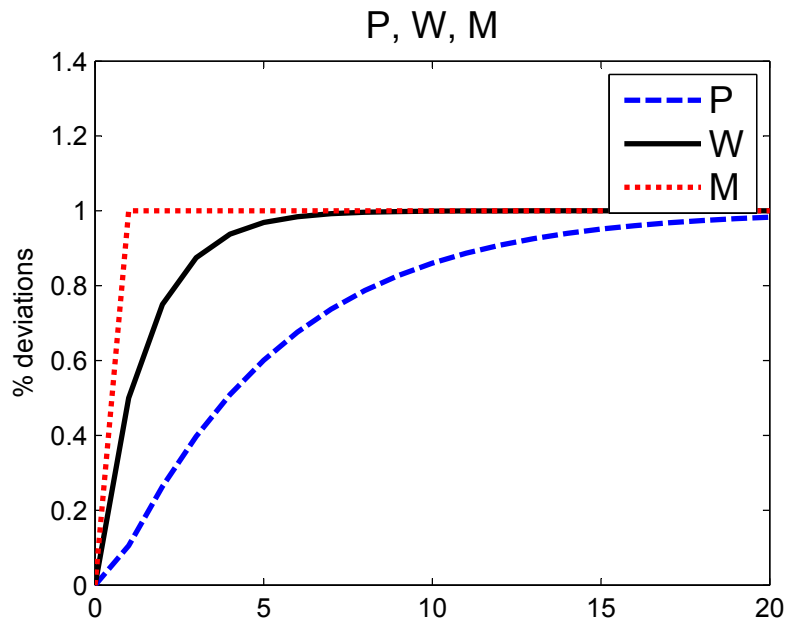




Figure 4: Simple Calvo, effect of depreciation rate on impulse responses

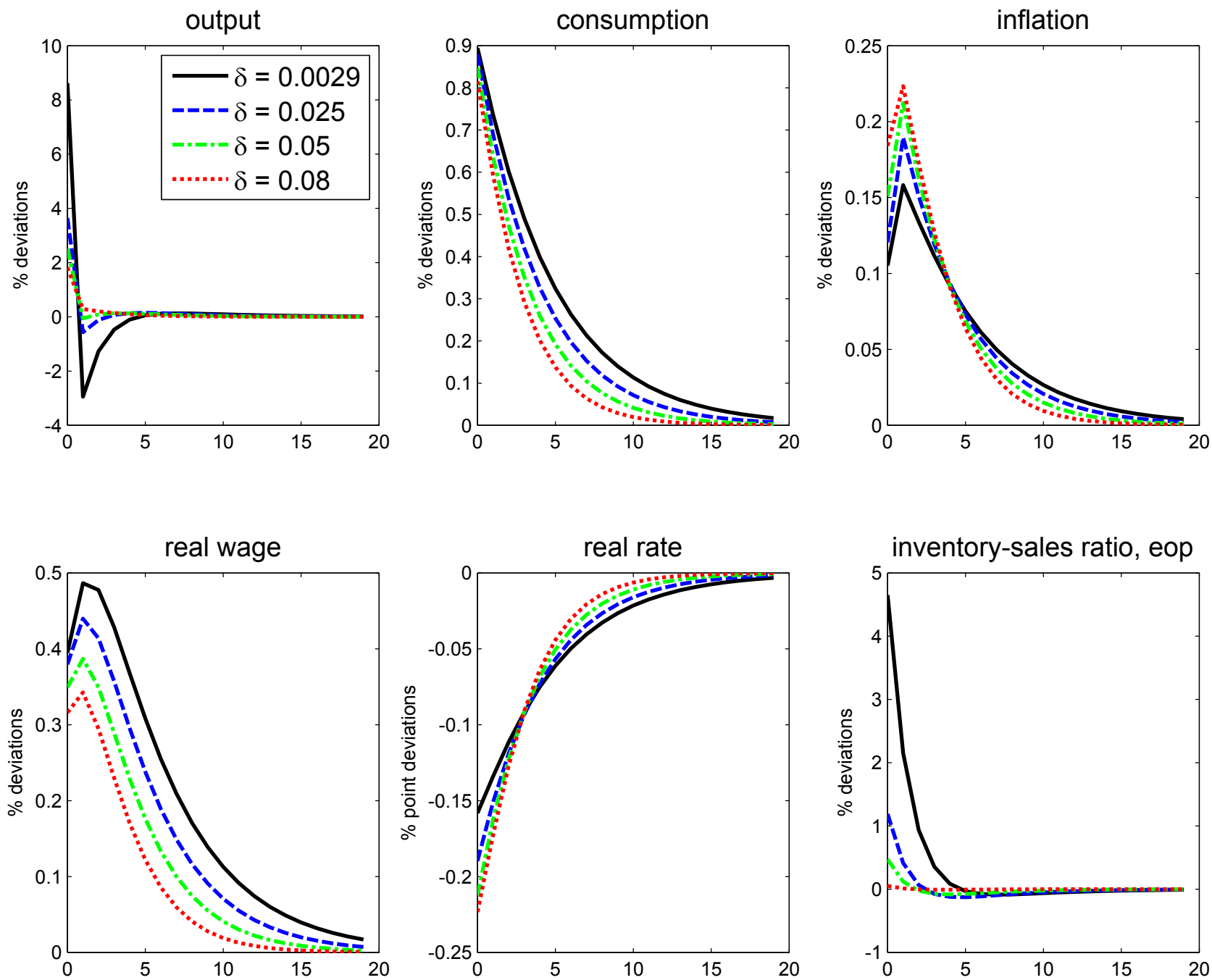


Figure 5: Simple Calvo with costs of output deviations from SS

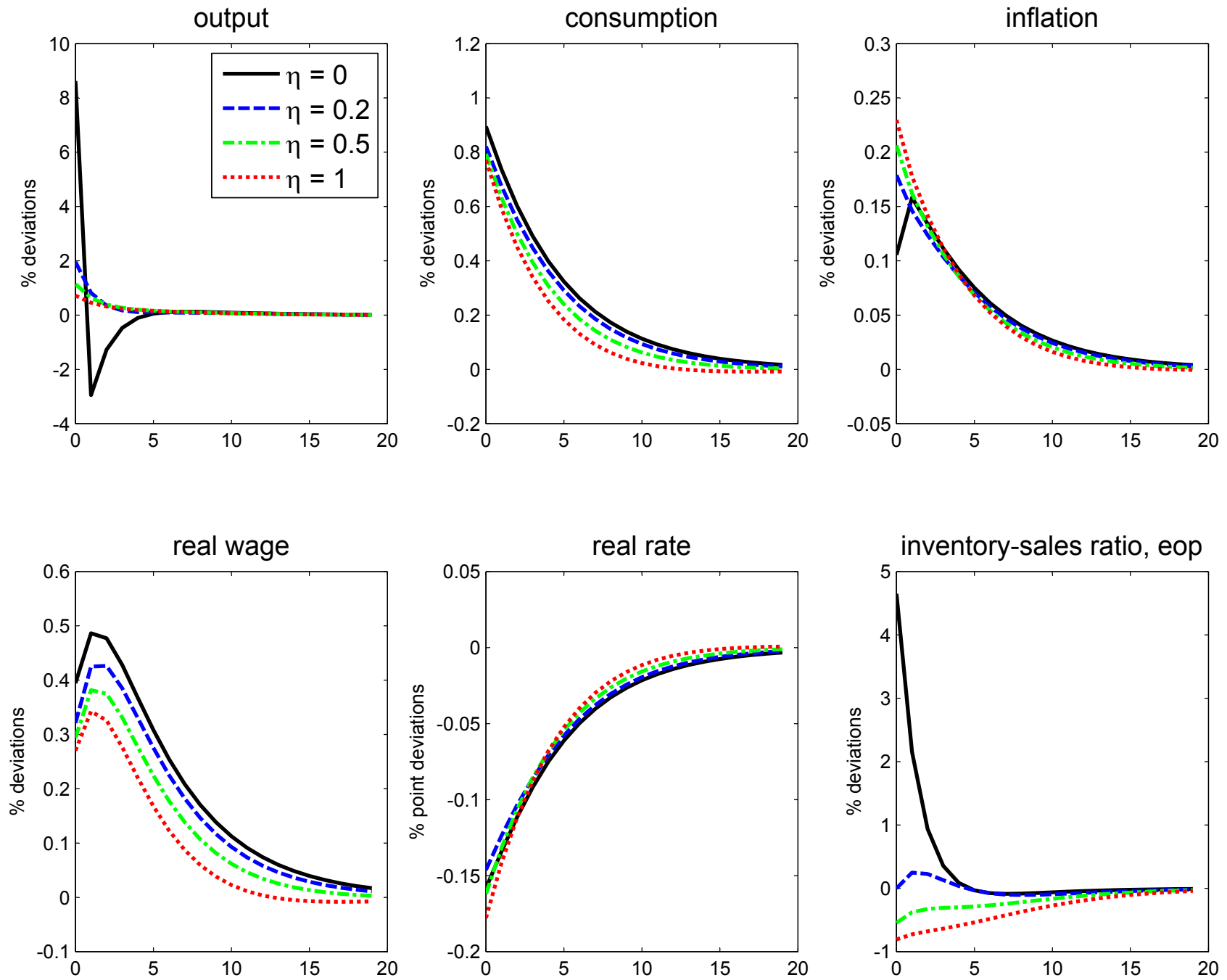


Figure 6: Simple Calvo with convex cost of changing output

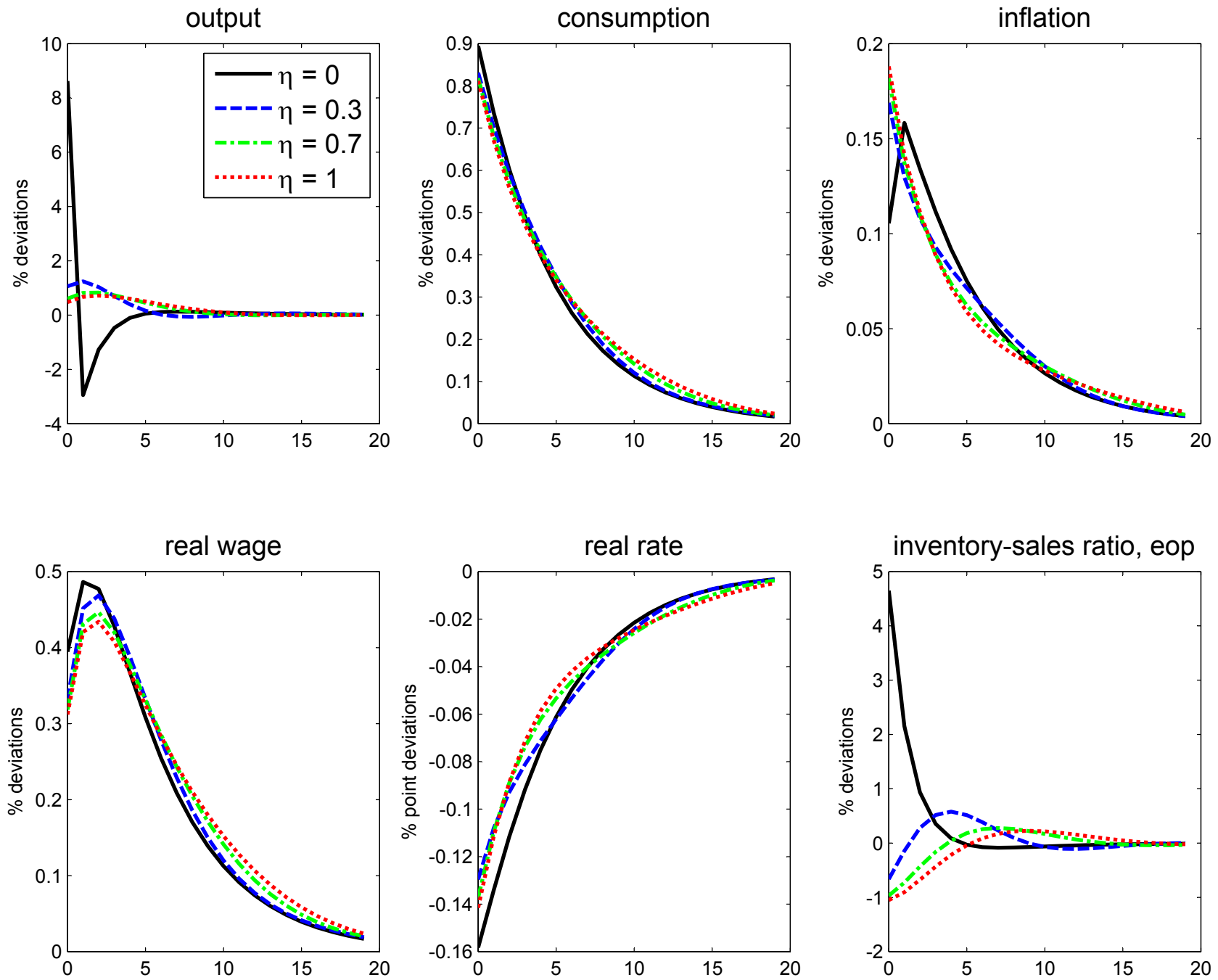


Figure 7: Larger real rigidities and stock adjustment cost

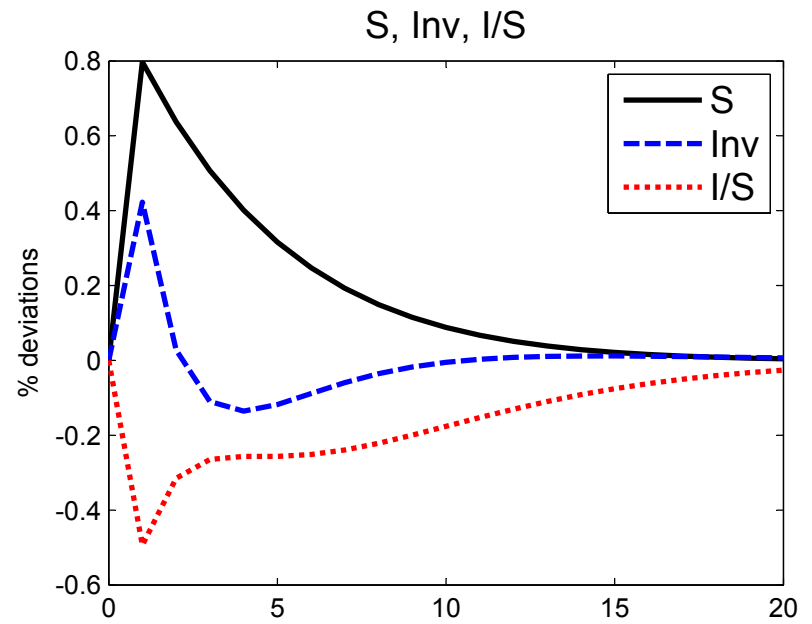
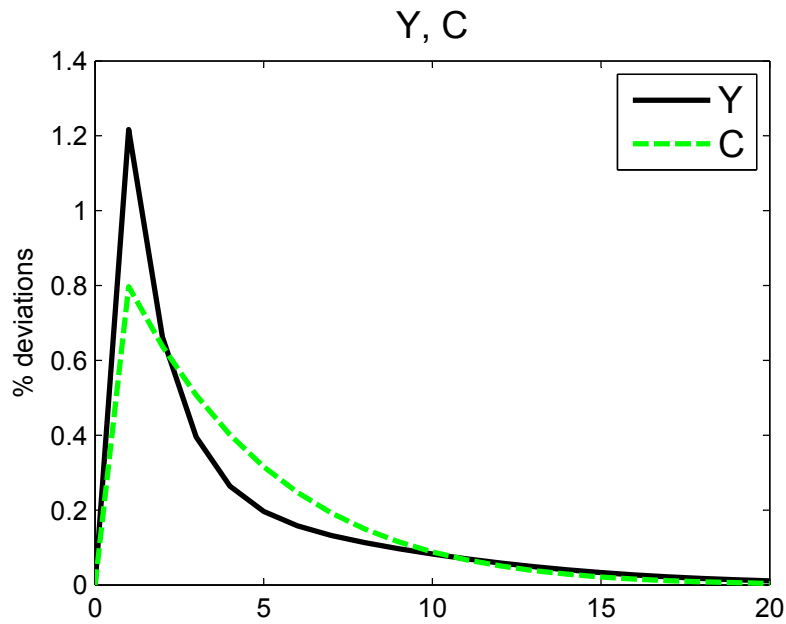
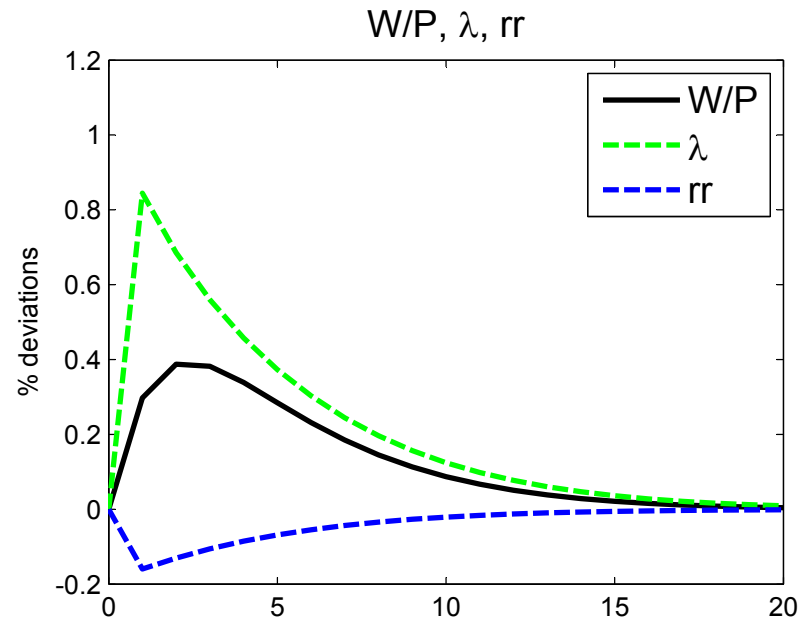
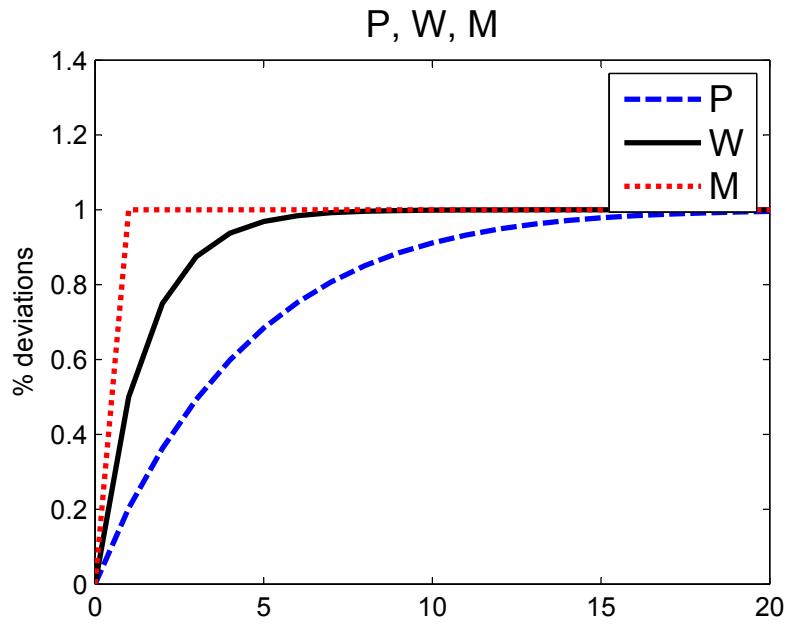


Figure 8: Imposing paths for real wage and real interest rate from data

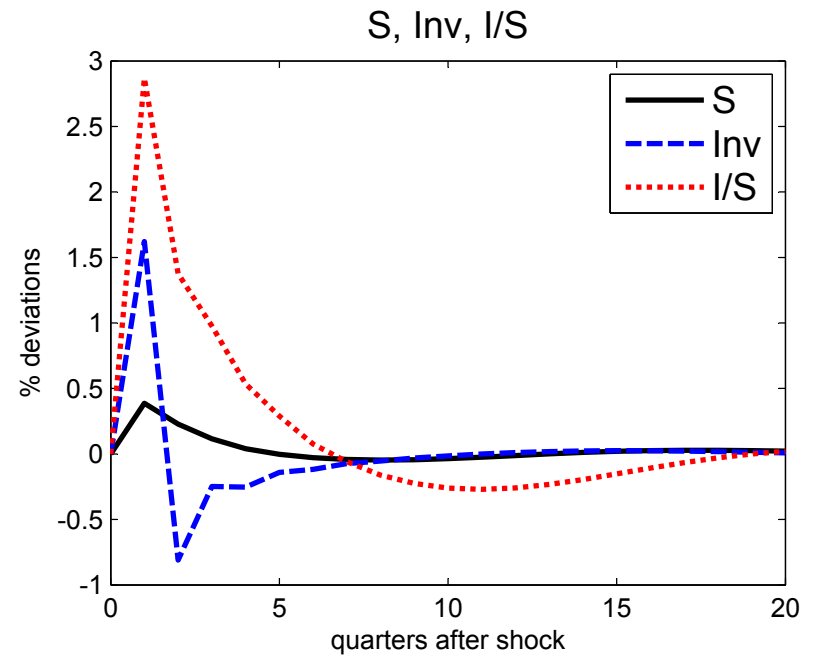
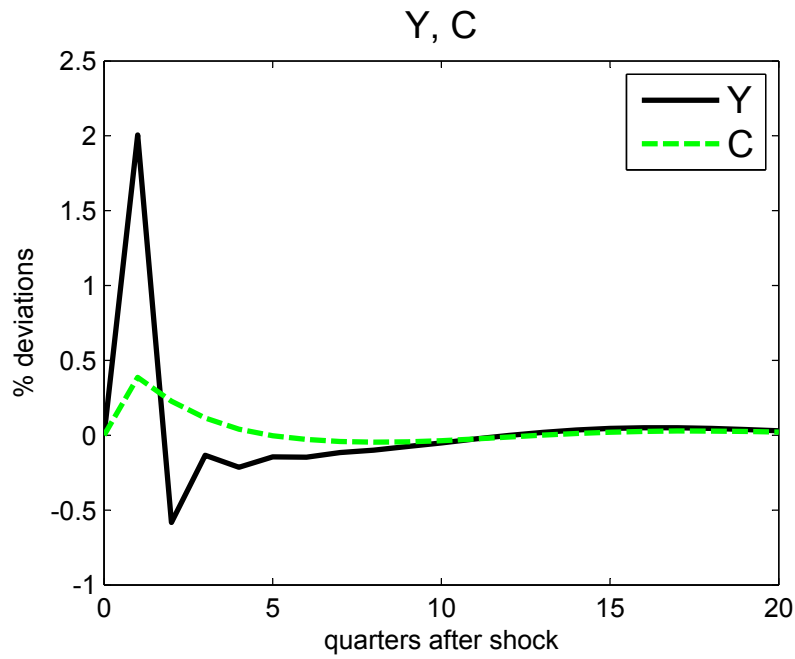
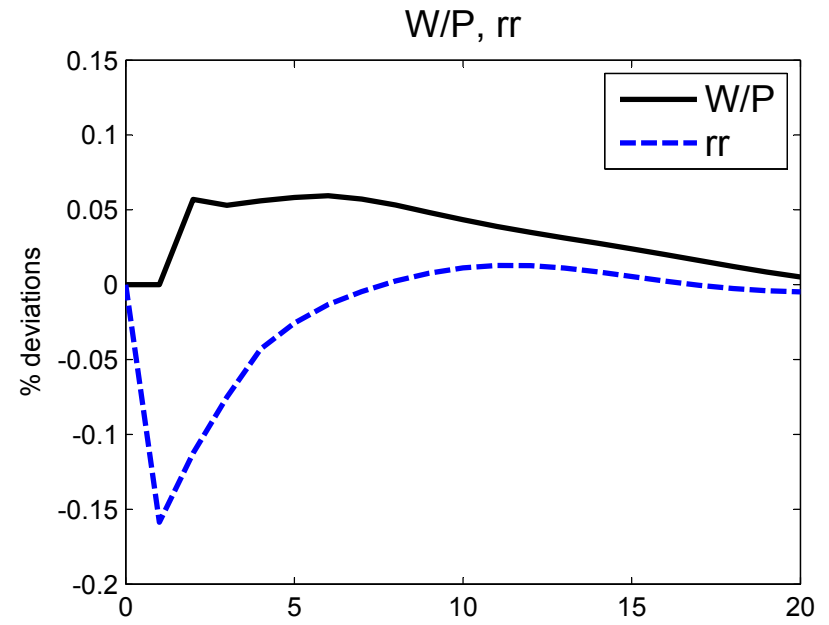
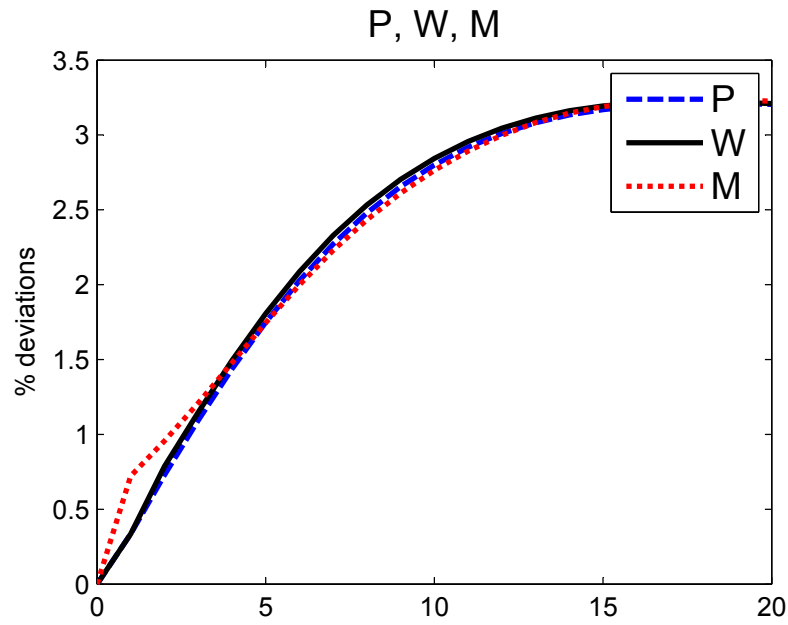


Figure 9: SW(07) with inventories, response to expansionary R shock

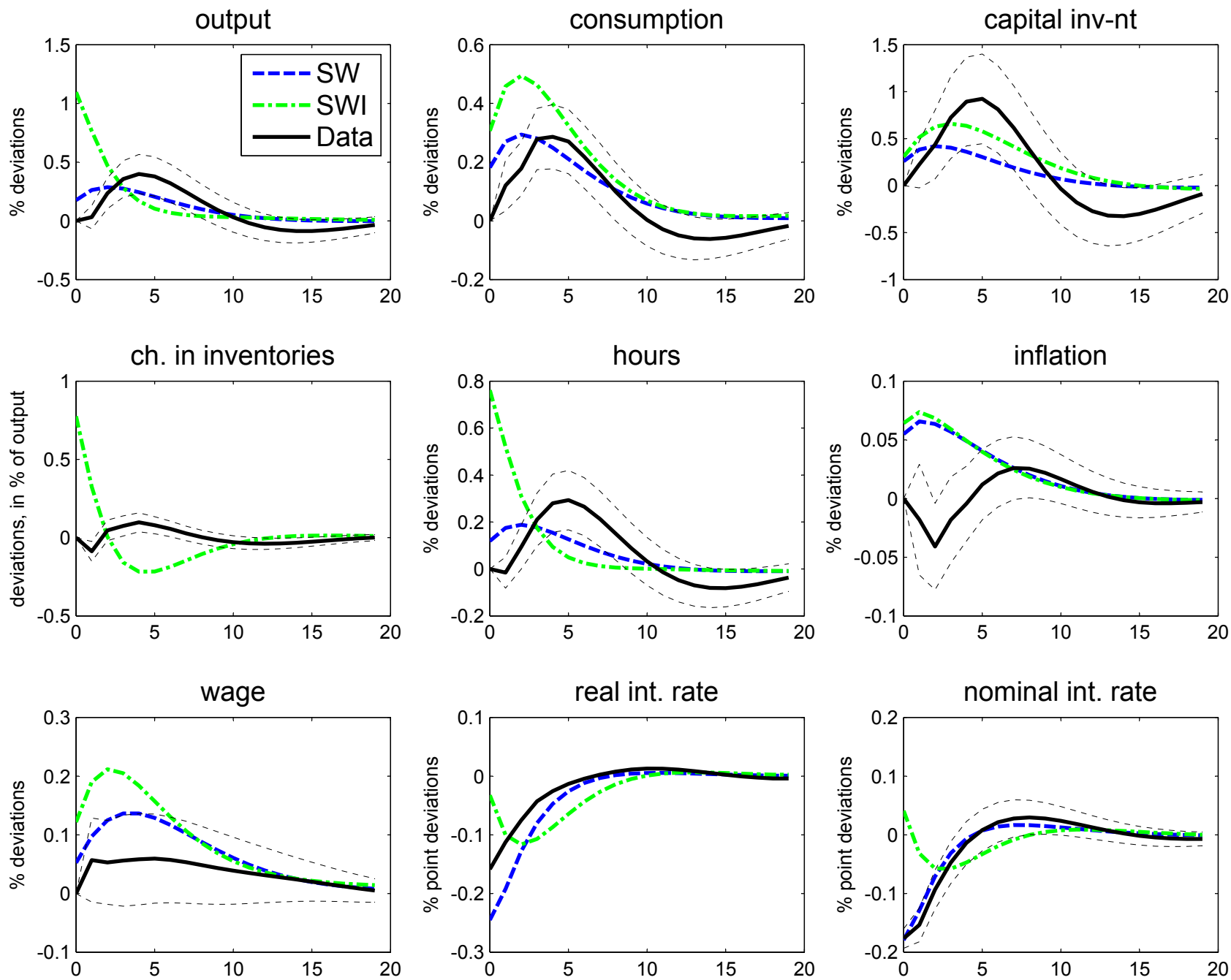


Figure 10: Smets-Wouters (2007) with inventories for different rates of depreciation

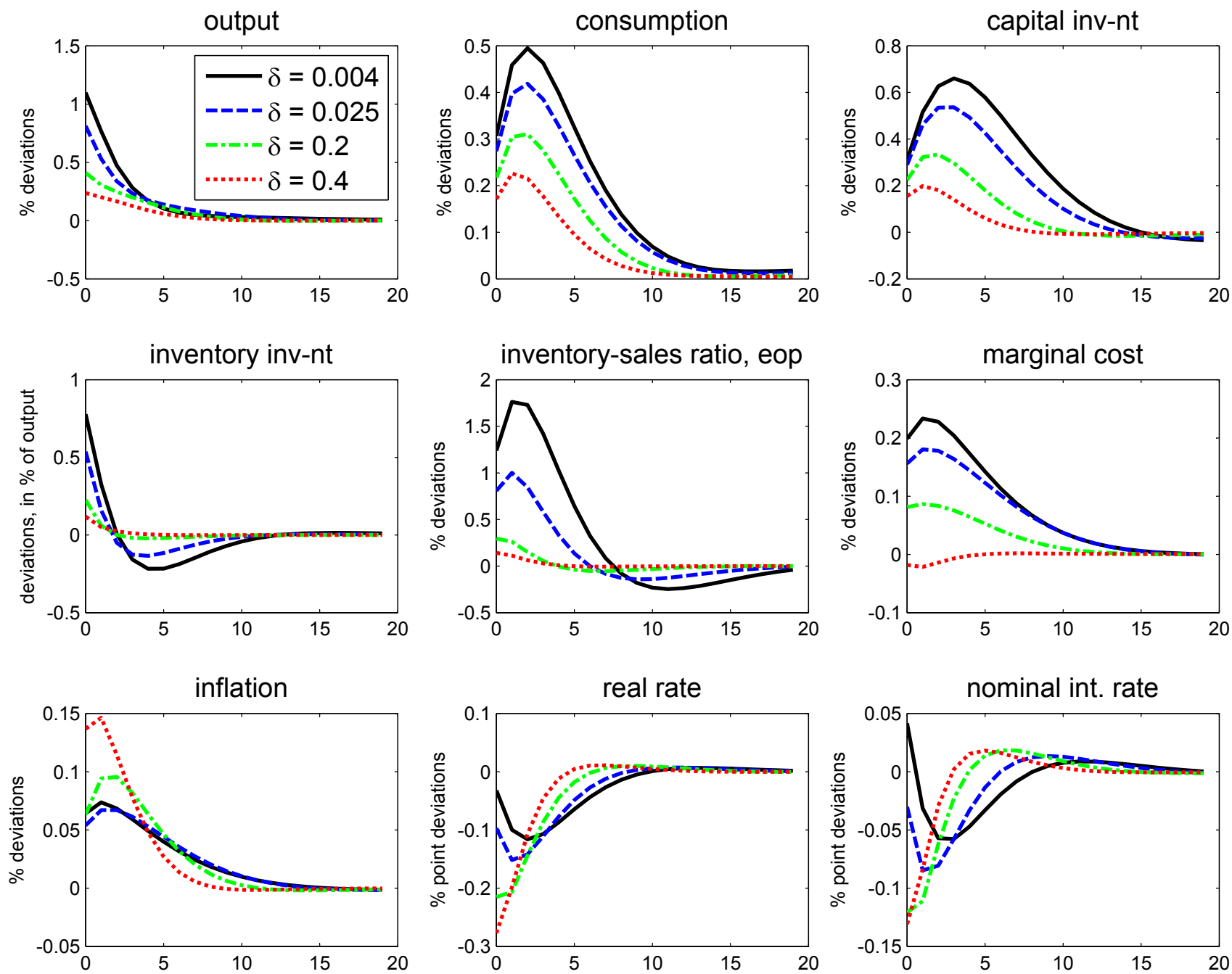


Figure 11: Smets and Wouters (2007) with inventories, different adj. costs

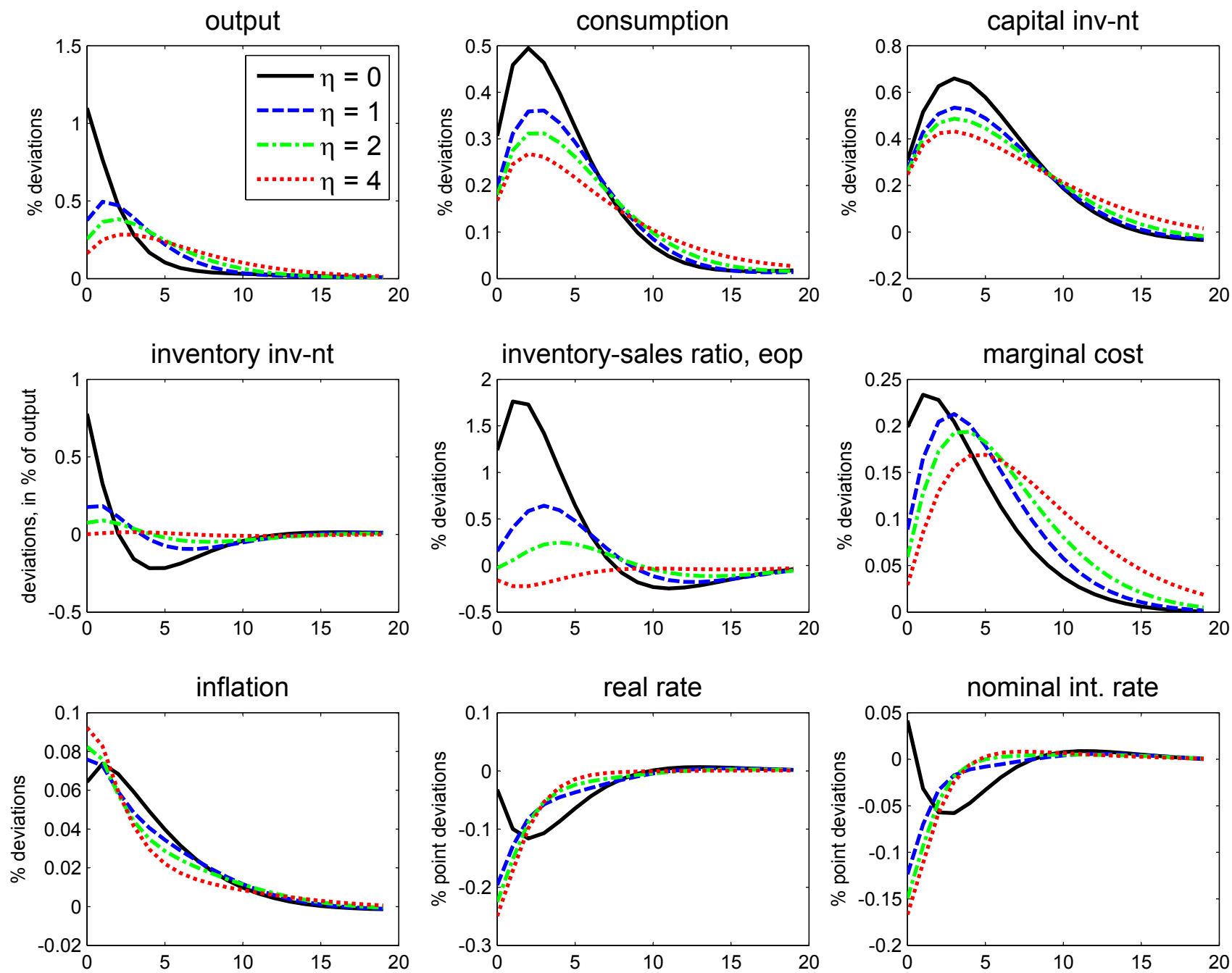




Figure 12: Reestimated SW(07) with inventories and adj. costs

